

Joint CTEQ Meeting and 7th International Conference on Physics Opportunities at an EIC (POETIC 7)

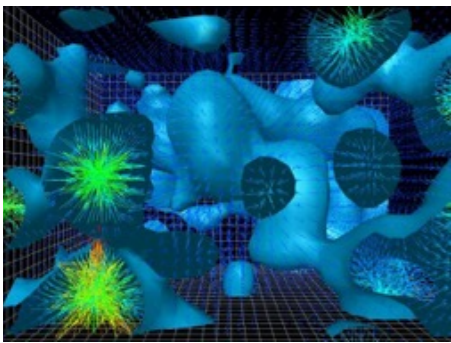
The Mass of the Nucleon

Martin J Savage

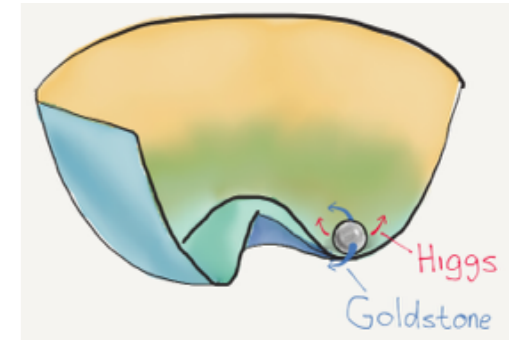


INSTITUTE for
NUCLEAR THEORY



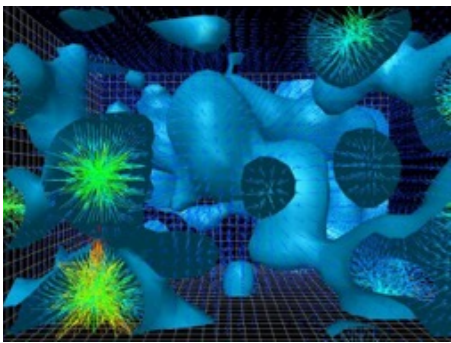


A Nice and Stimulating Workshop

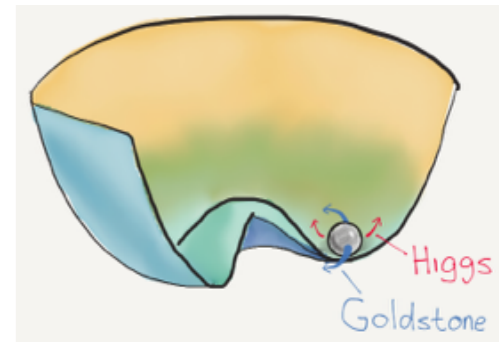


Much of what I present results from discussions from this meeting

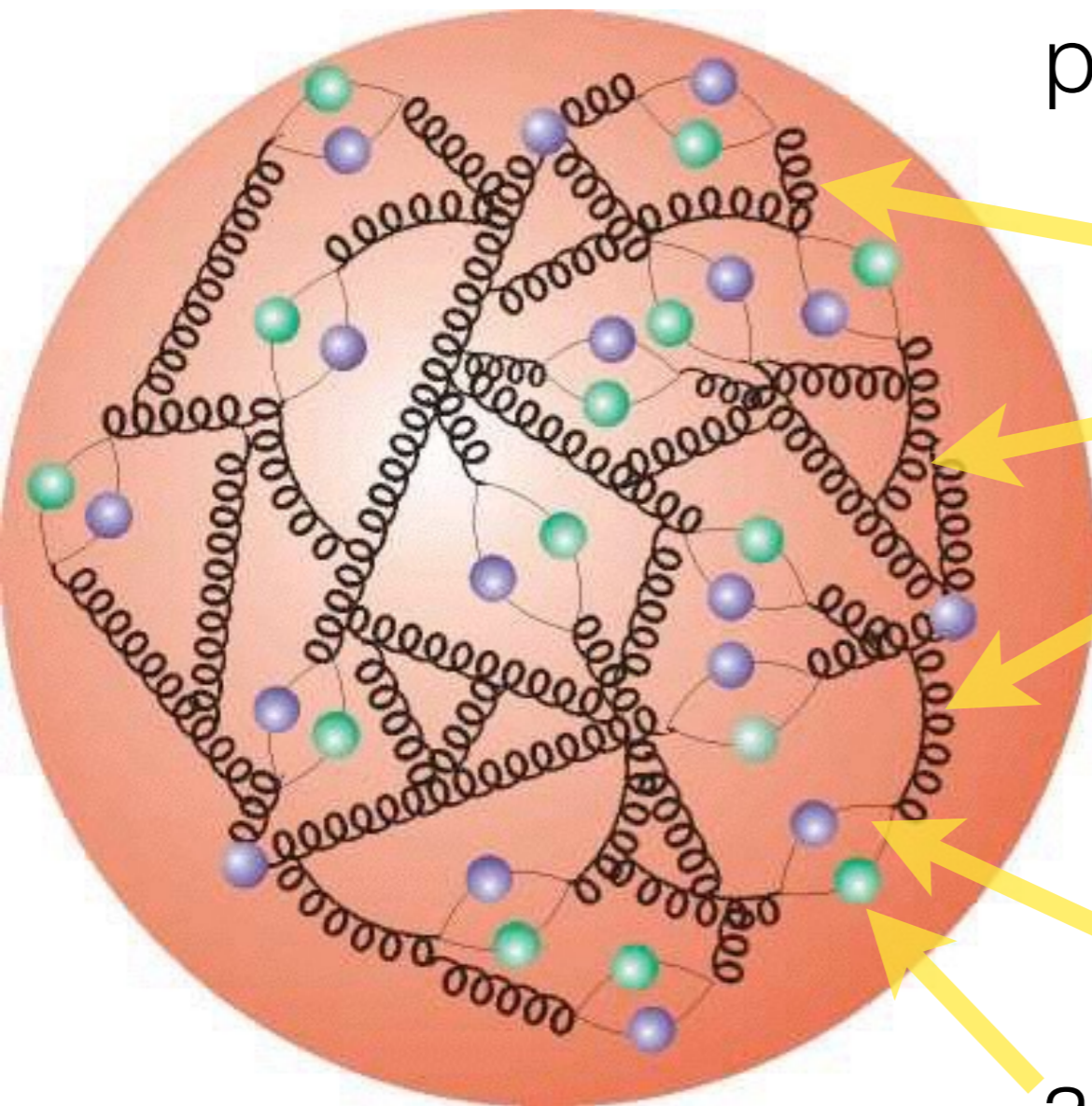
Stanley J. Brodsky
Leonard Gamberg
Xiangdong Ji
Sylvester Joosten
Dmitri Kharzeev
Keh-Fei Liu
Andreas Metz
Zein-Eddine Meziani
Alfred Mueller
Berndt Mueller
Jim Napolitano
Jianwei Qiu
David Richards
Craig Roberts
Martin Savage
Nikos Sparveris
Stepan Stepanyan
George Sterman
Bernd Surrow
Yi-Bo Yang



Masses of Strongly Interacting Particles



proton mass = $938.272046(21)$ MeV



gluon

quark

anti-quark

Nucleon

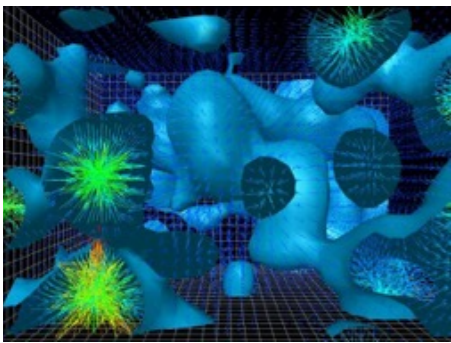
up-quark mass ~ 2.2 MeV

down-quark mass ~ 4.7 MeV

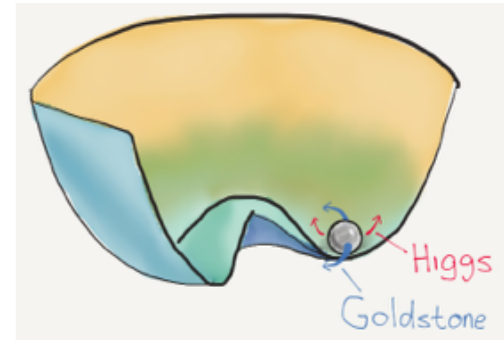
gluon mass = 0.0 MeV

Nucleon is an entangled state of indefinite particle number with spin-1/2

quark masses defined in a scheme at a scale:
e.g. Dim. Reg. with $\overline{\text{MS}}$ at $\mu=2$ GeV



Origins in the Standard Model

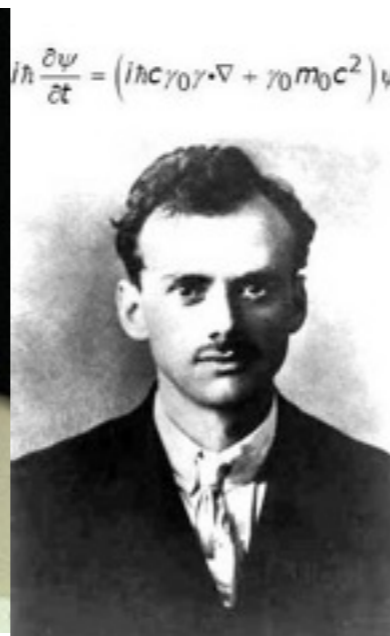


$G_{\mu\nu}G^{\mu\nu}$, $\bar{q} \gamma_\mu D^\mu q$
Gluons and quark kinetic

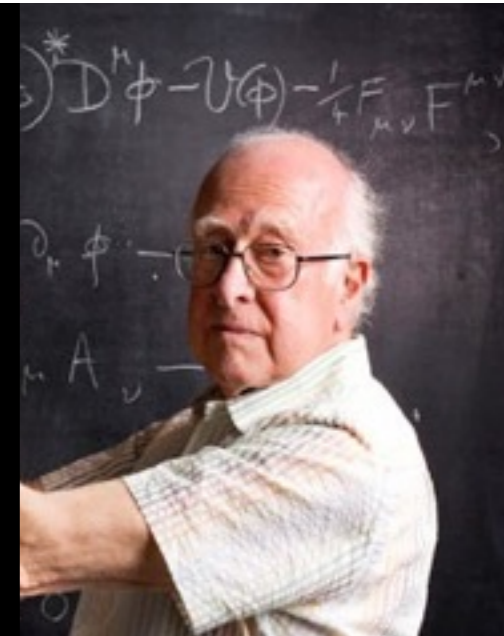
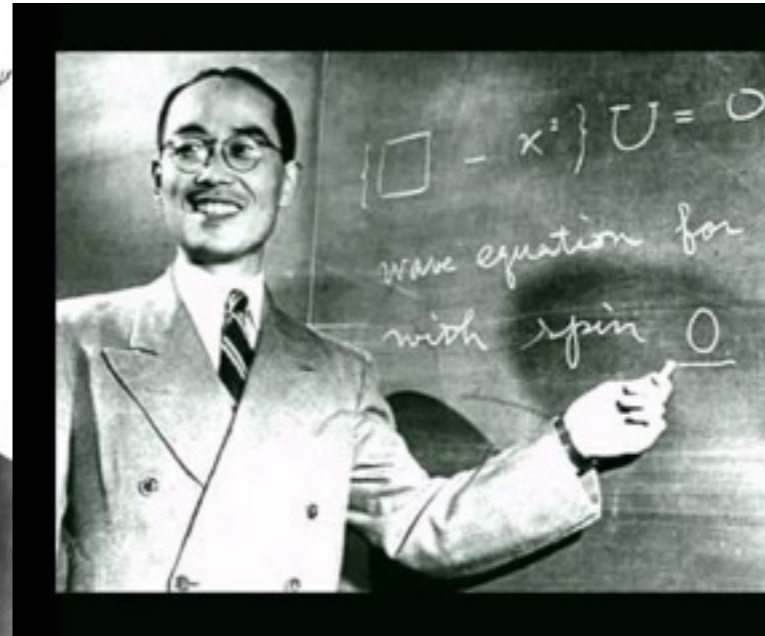
$g^{ij} \bar{q}_i \not{v} q_j$
quark masses

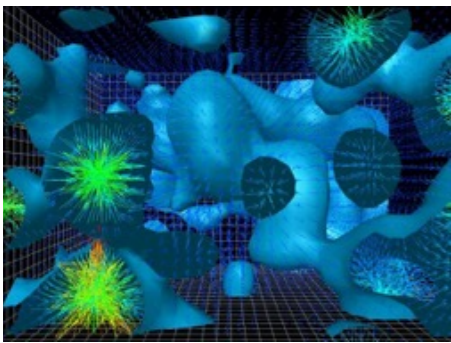
$F_{\mu\nu}F^{\mu\nu}$
E+M

Yang-Mills and Dirac

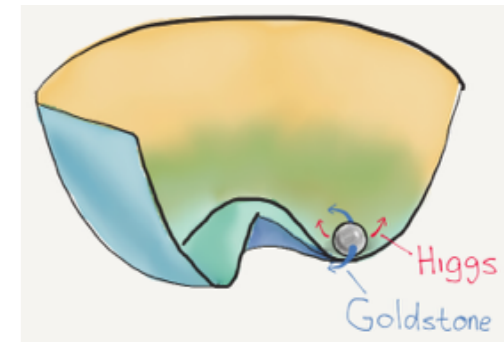


Yukawa and Higgs

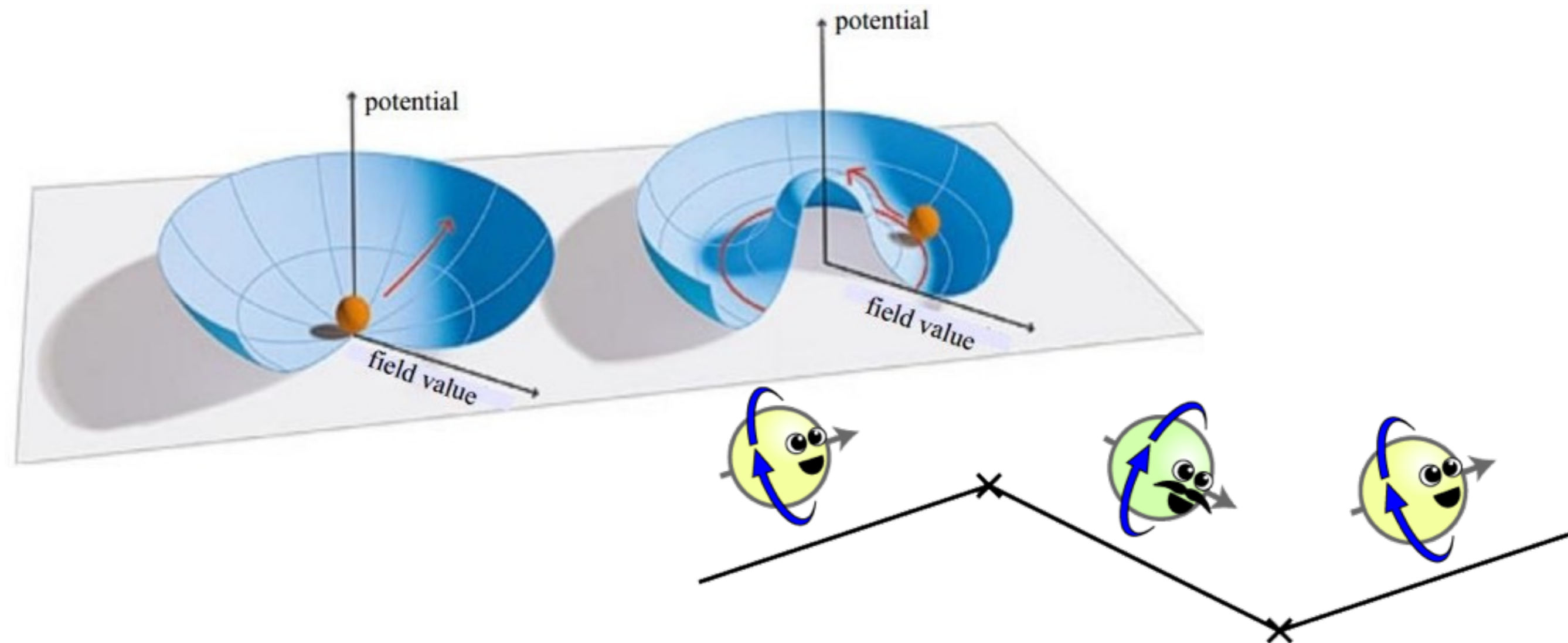




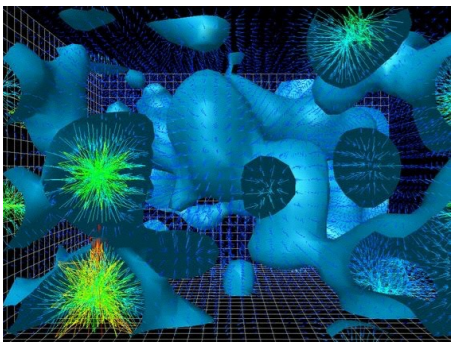
Two ``Nobel-Prize'' Contributions



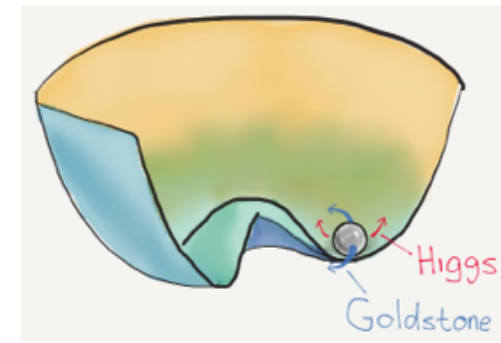
The Higgs Mechanism - UV Physics



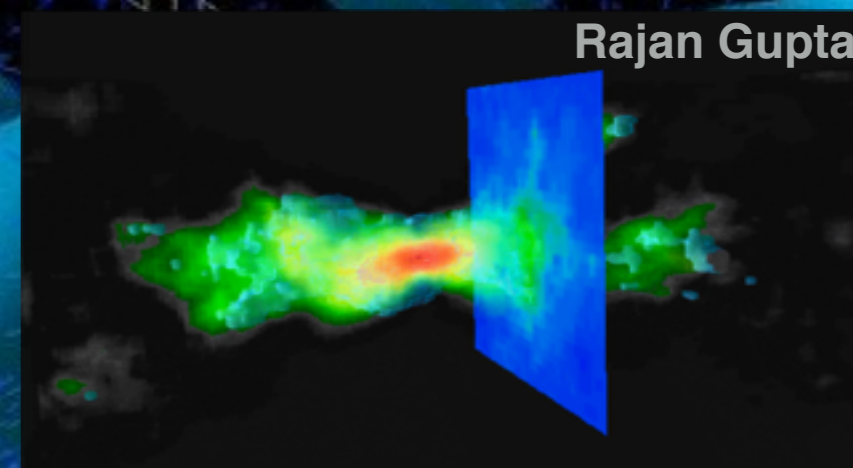
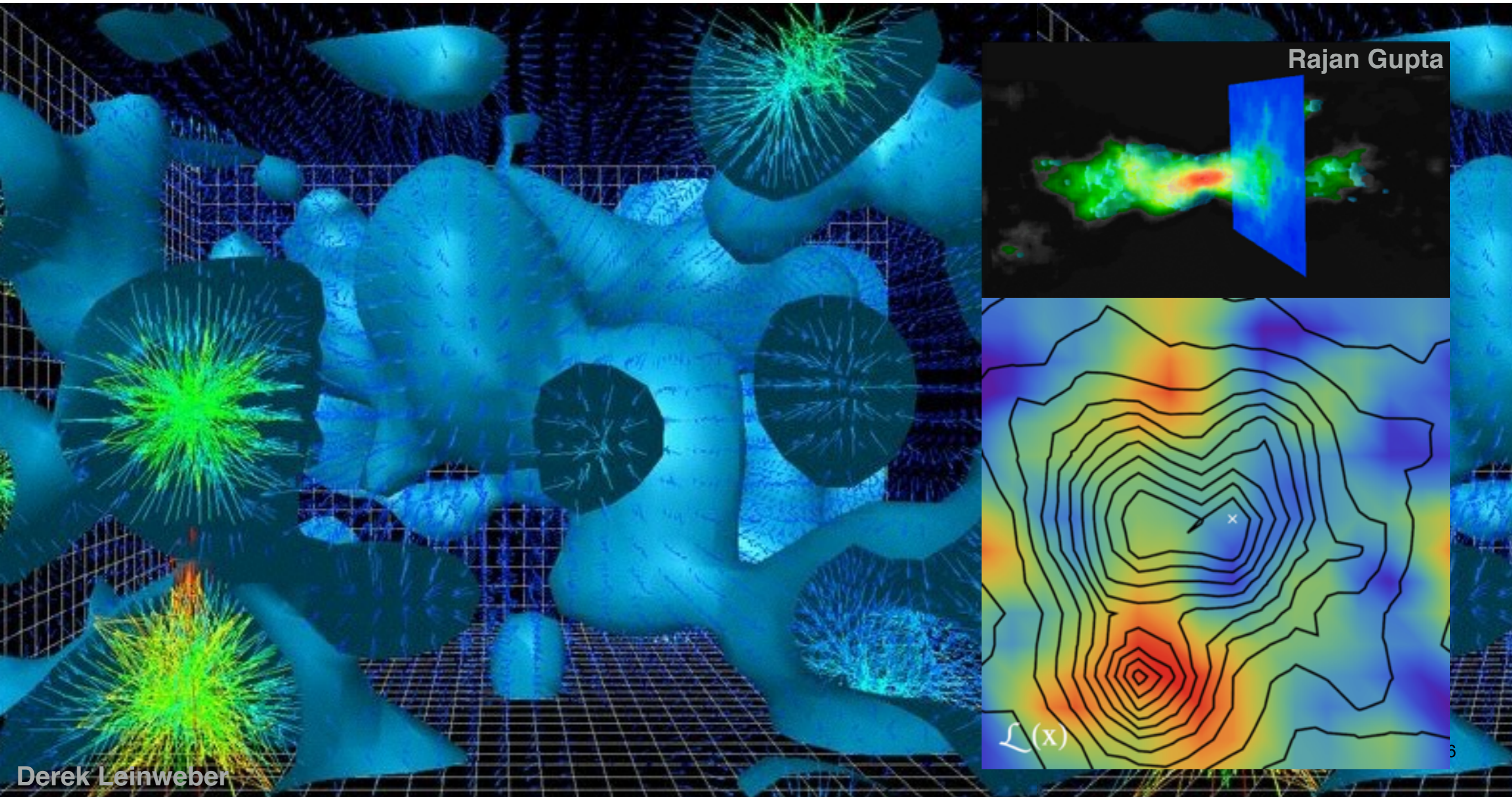
Massless matter fields of the Standard Model endowed with mass through spontaneous symmetry breaking at the electroweak scale



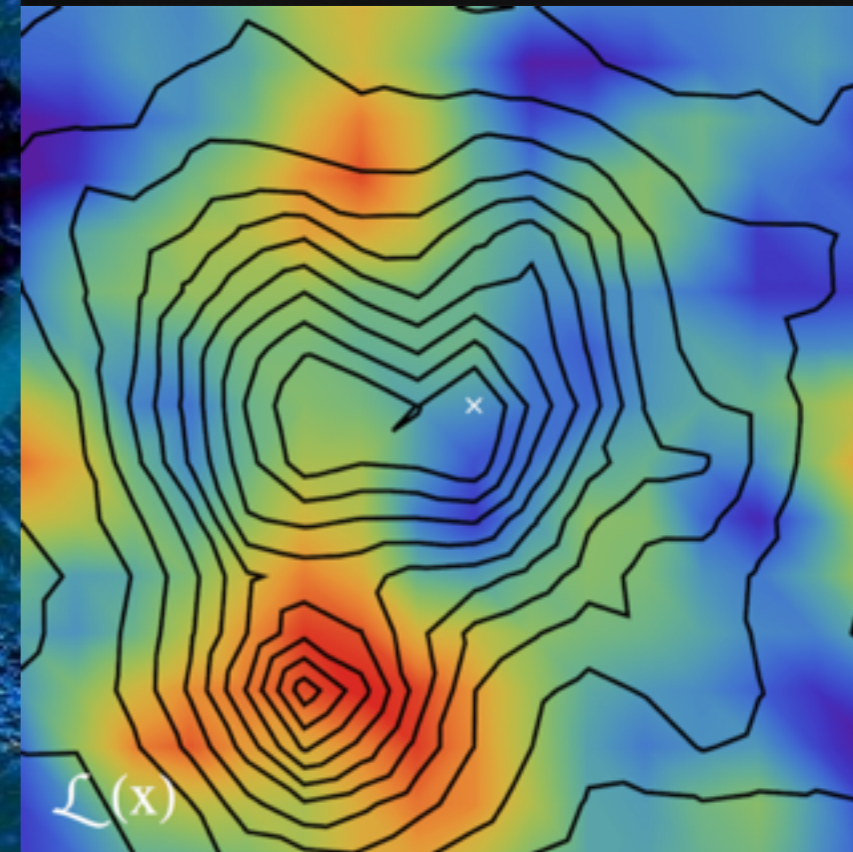
Two ``Nobel-Prize'' Contributions

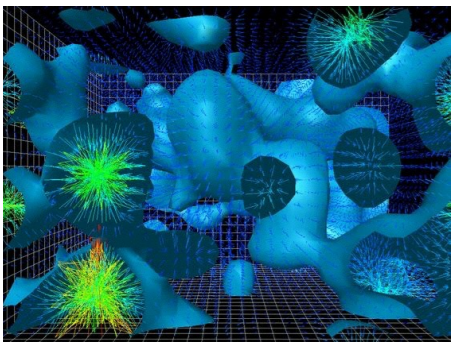


Strong Quantum Fluctuations - IR Physics

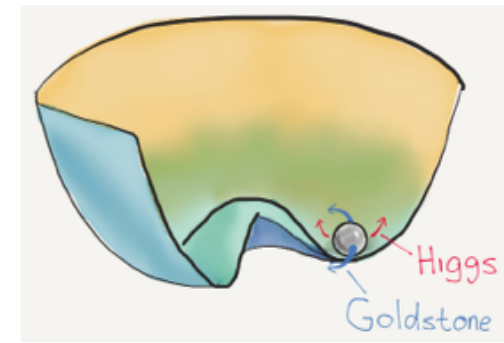


Rajan Gupta

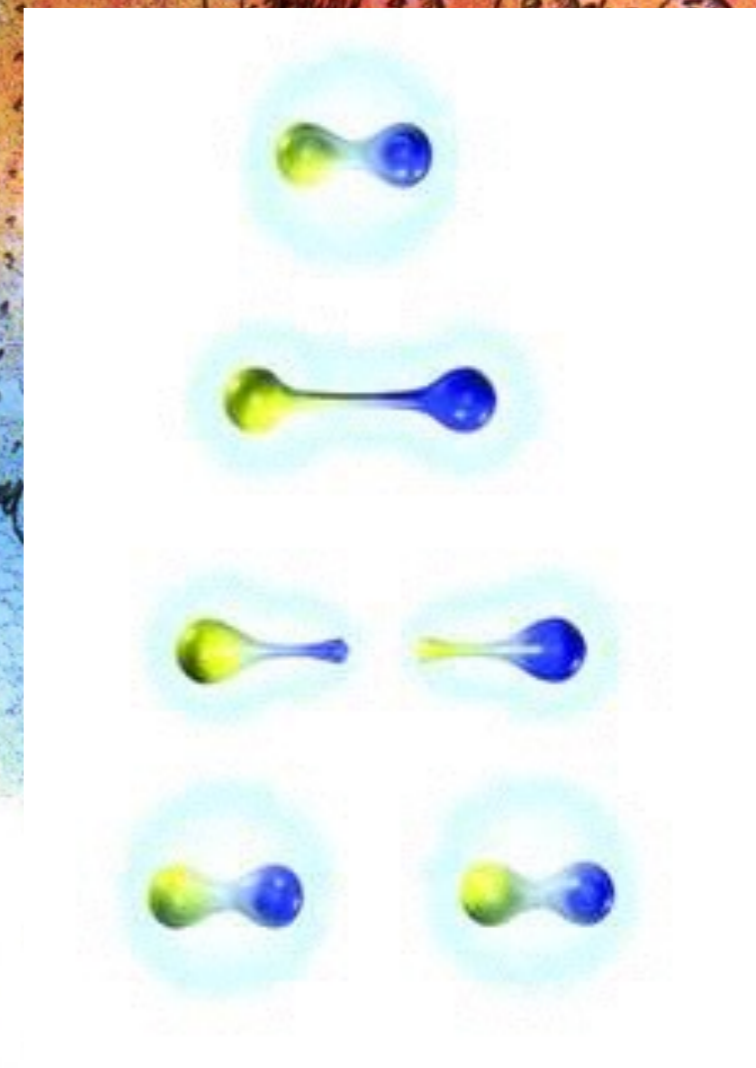
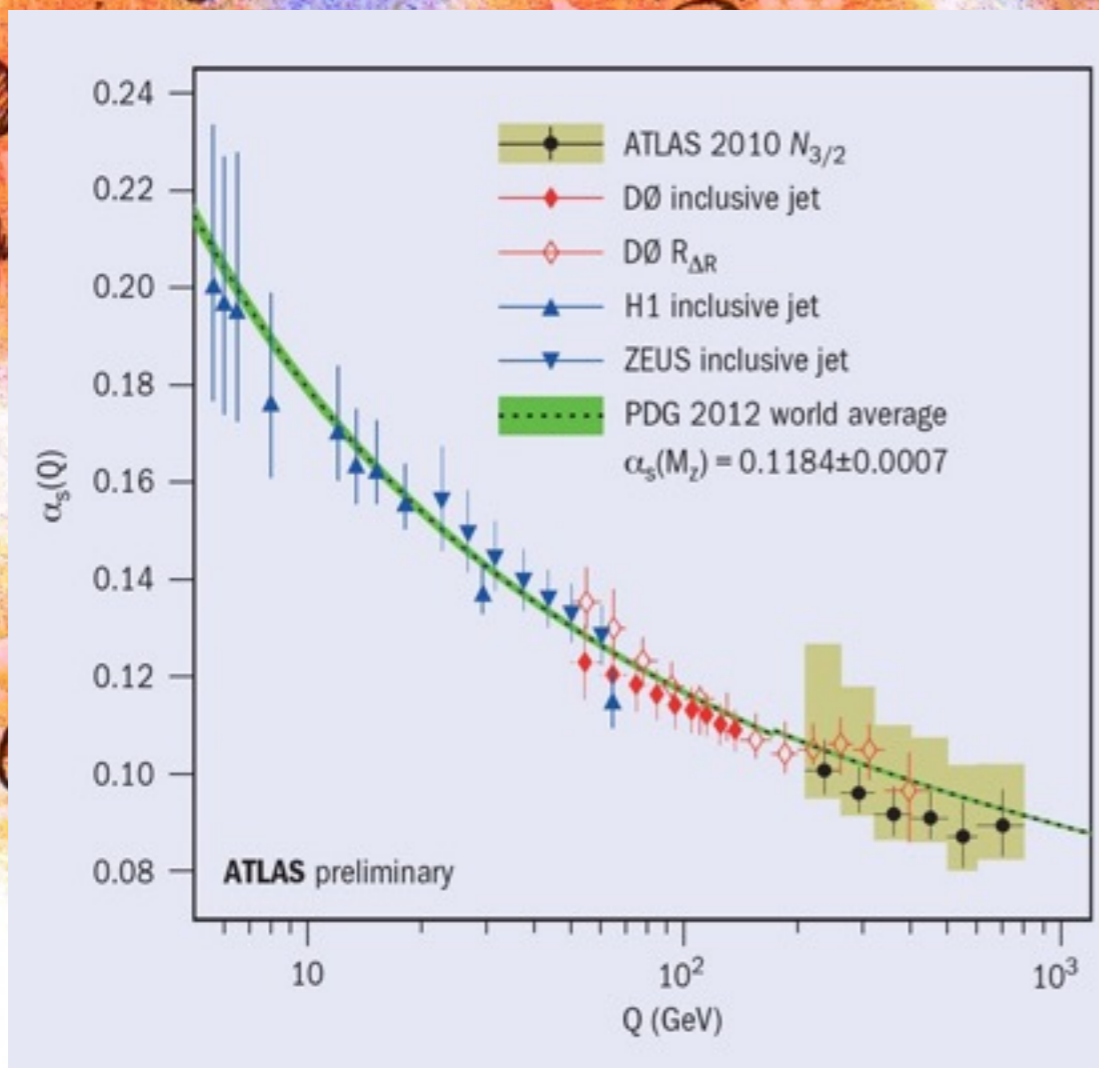




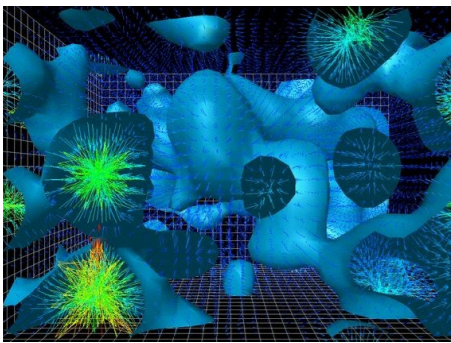
Two ``Nobel-Prize'' Contributions



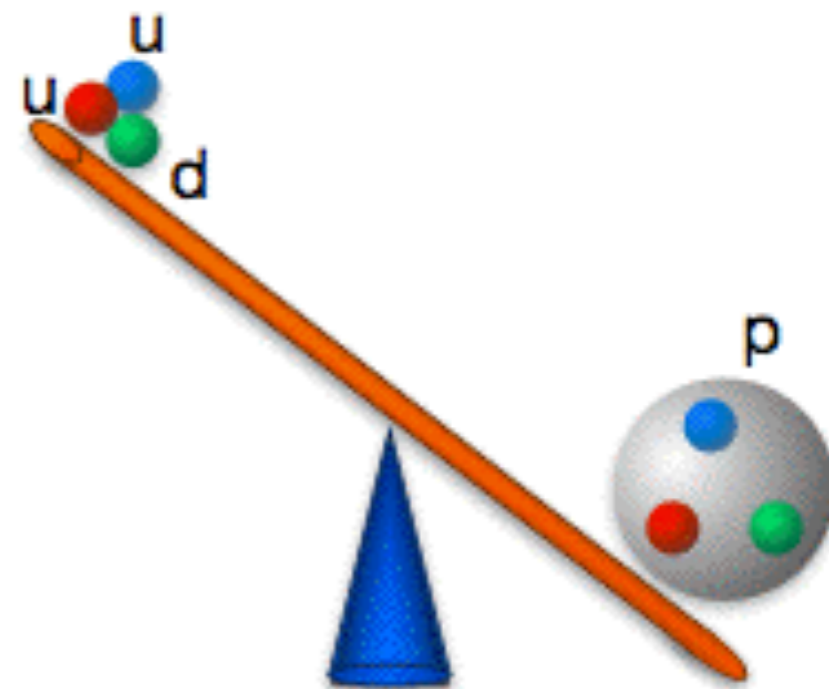
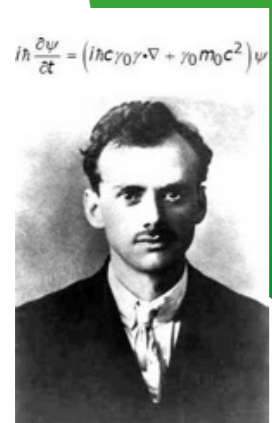
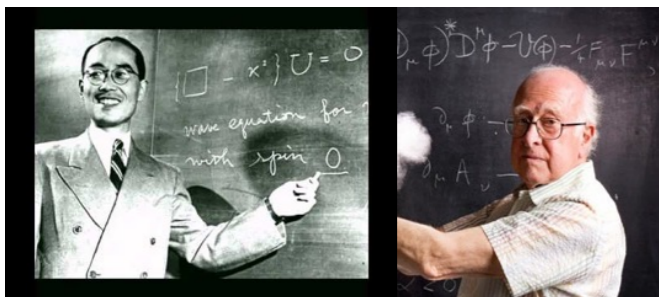
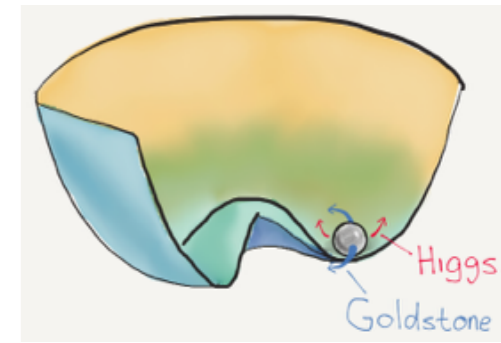
Strong Quantum Fluctuations - IR Physics



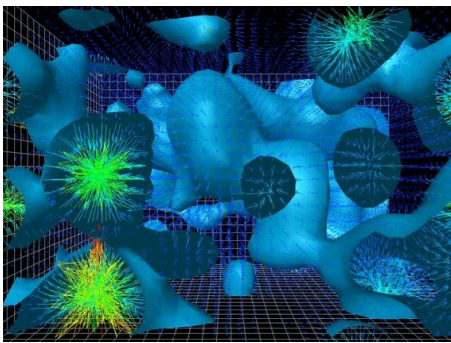
Asymptotic Freedom, Confinement, Chiral Symmetry Breaking



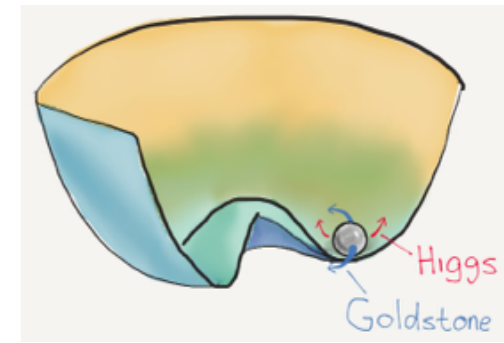
Hadron Masses Decomposition ?



- A meaningful decomposition ?
- What do we currently know ?
- How to probe and constrain further ?
- Are there fine-tunings ?

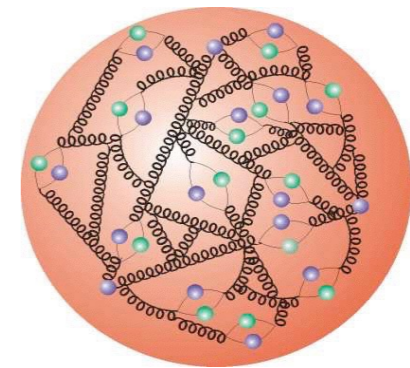


Hadron Mass Decomposition



$$\mathcal{L} = \underbrace{\frac{1}{4} G^{\mu\nu} G_{\mu\nu}}_{\text{QCD}} + \bar{q} (\not{D} + m_q) q + \underbrace{\frac{1}{4} F^{\mu\nu} F_{\mu\nu}}_{\text{QED}}$$

Quarks and Gluons

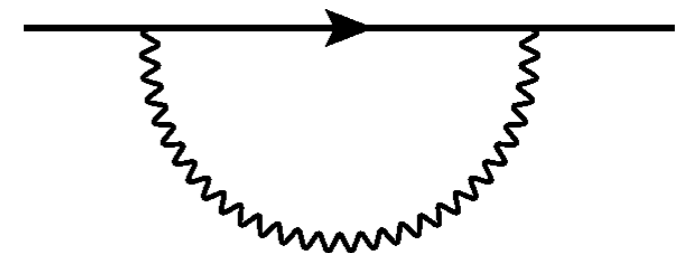


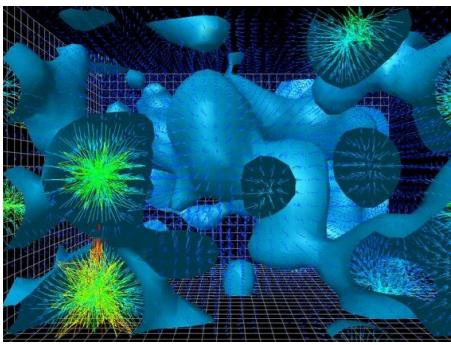
Equals

$$\mathcal{L} = \bar{N} (\not{D} + M_N) N + \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

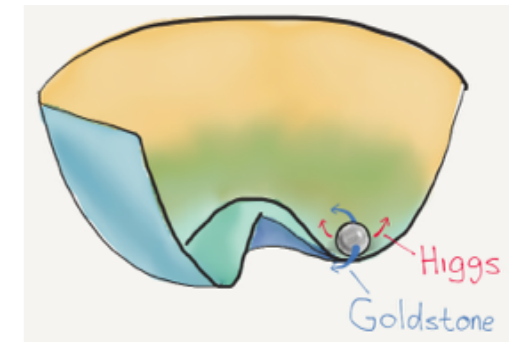
EFT

Nucleon EFT





Chiral Symmetry Results - quark masses

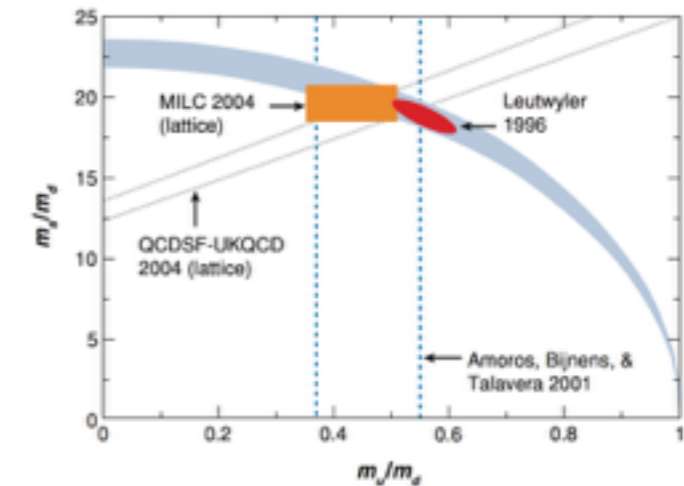


Chiral Perturbation Theory

Véronique Bernard¹ and Ulf-G. Meißner²

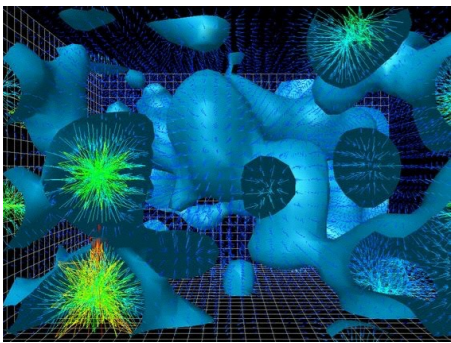
¹Laboratoire de Physique Théorique, Université Louis Pasteur, F-67084 Strasbourg, France; email: bernard@lpt6.u-strasbg.fr

²Helmholtz Institut für Strahlen und Kernphysik, Universität Bonn, D-53115 Bonn, Germany; and Forschungszentrum Jülich, Institut für Kernphysik, D-52425 Jülich, Germany; email: meissner@itkp.uni-bonn.de

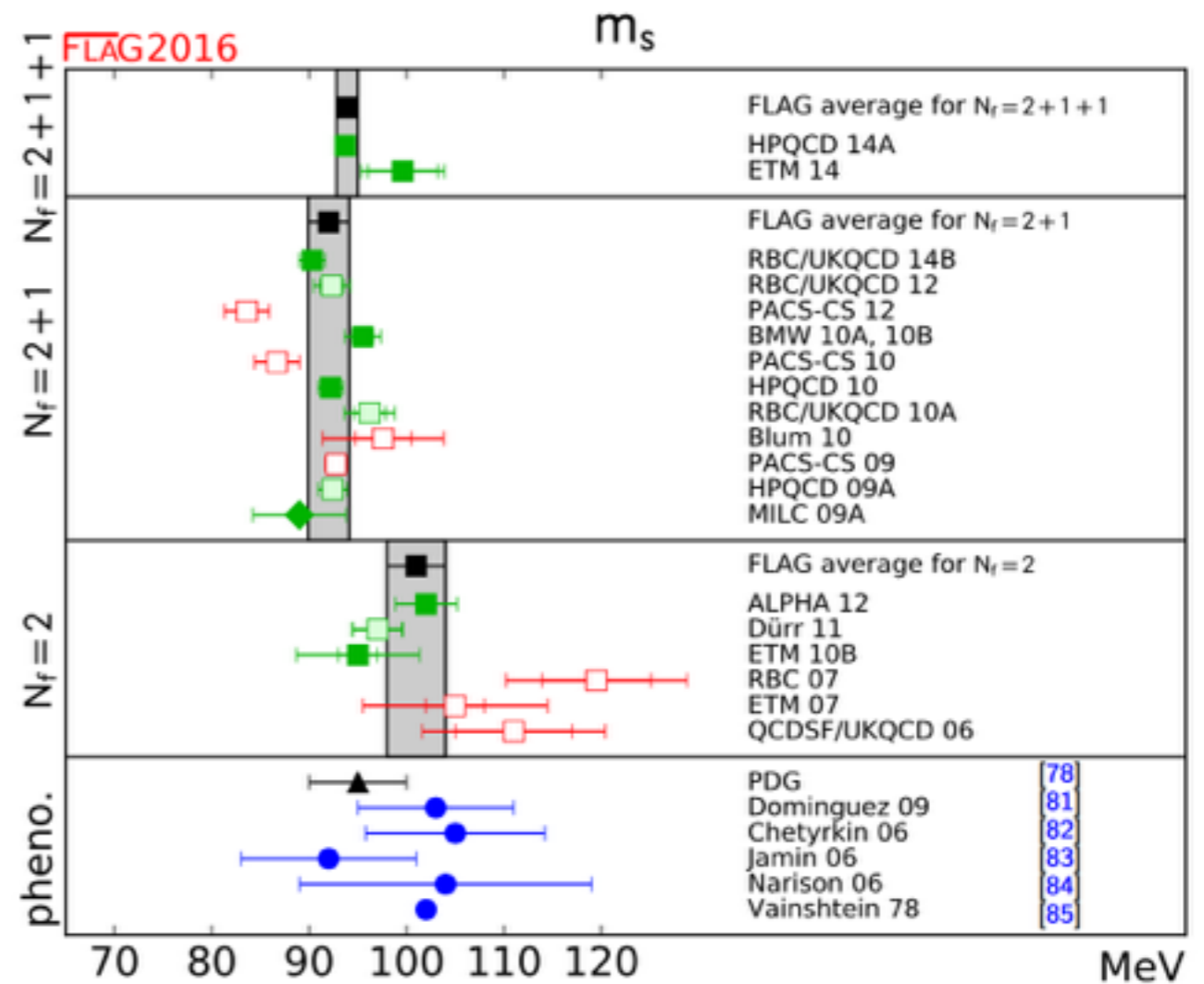
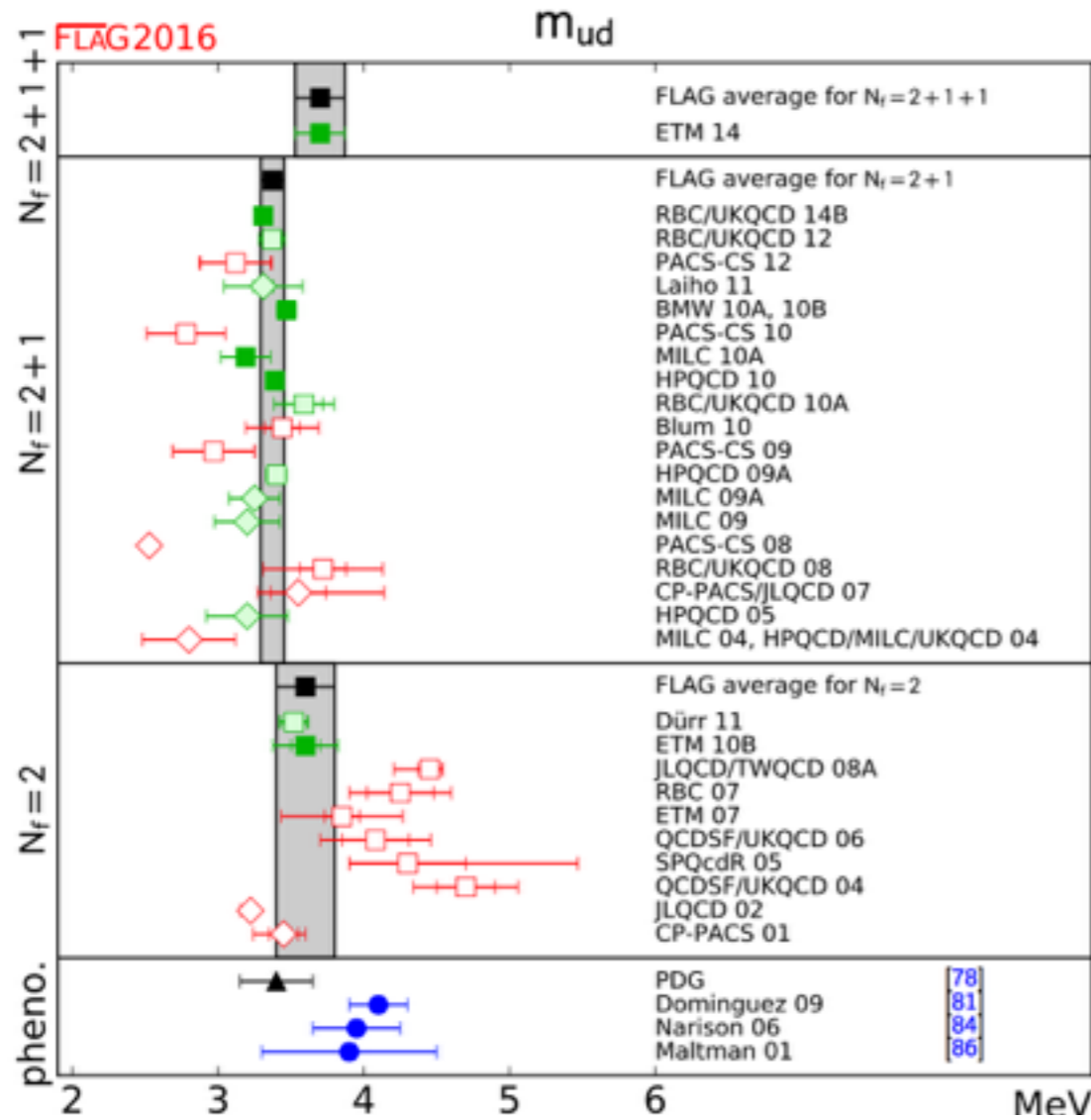
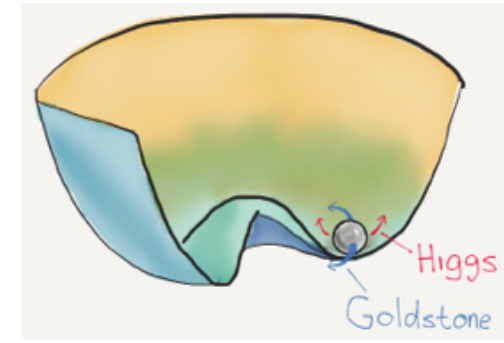


$$\frac{m_u}{m_d} = 0.553 \pm 0.043, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8, \quad \frac{m_s}{\hat{m}} = 24.4 \pm 1.5.$$

Leutwyler (1996)

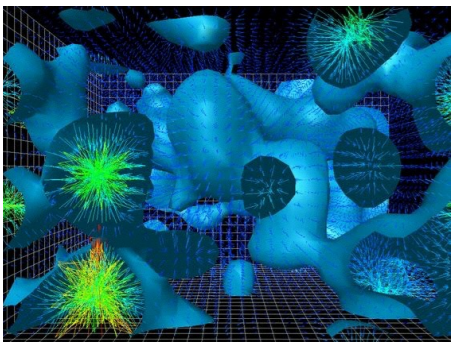


Lattice QCD: Results - quark masses

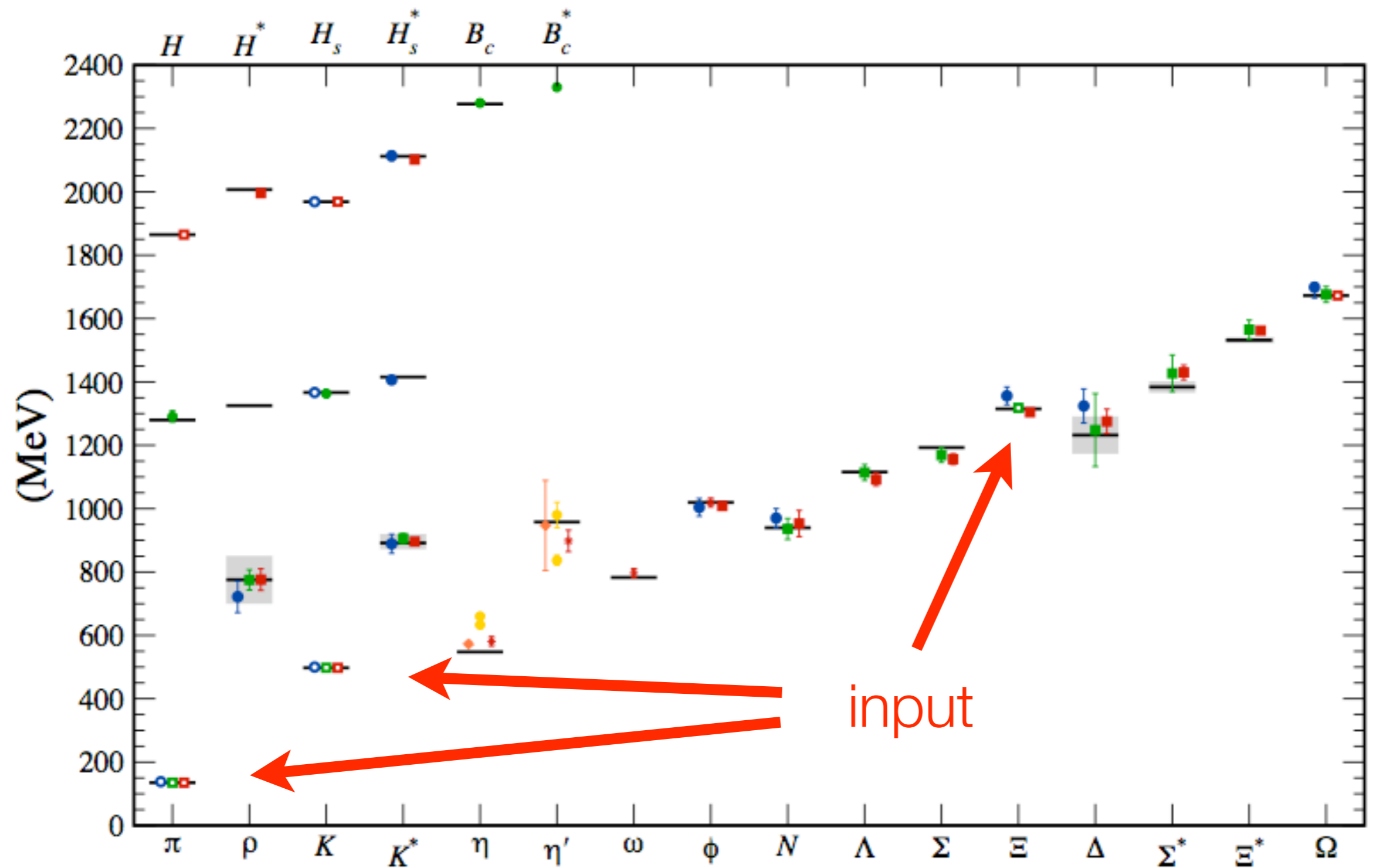
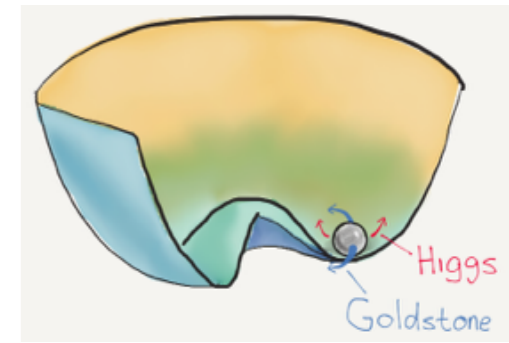


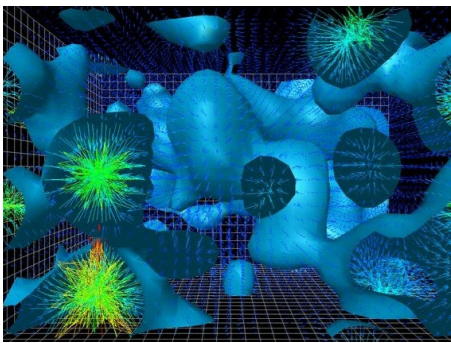
N_f	m_u	m_d	m_u/m_d
2+1+1	2.36(24)	5.03(26)	0.470(56)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)
2	2.40(23)	4.80(23)	0.50(4)

$$\overline{\text{MS}}, \mu = 2 \text{ GeV}$$

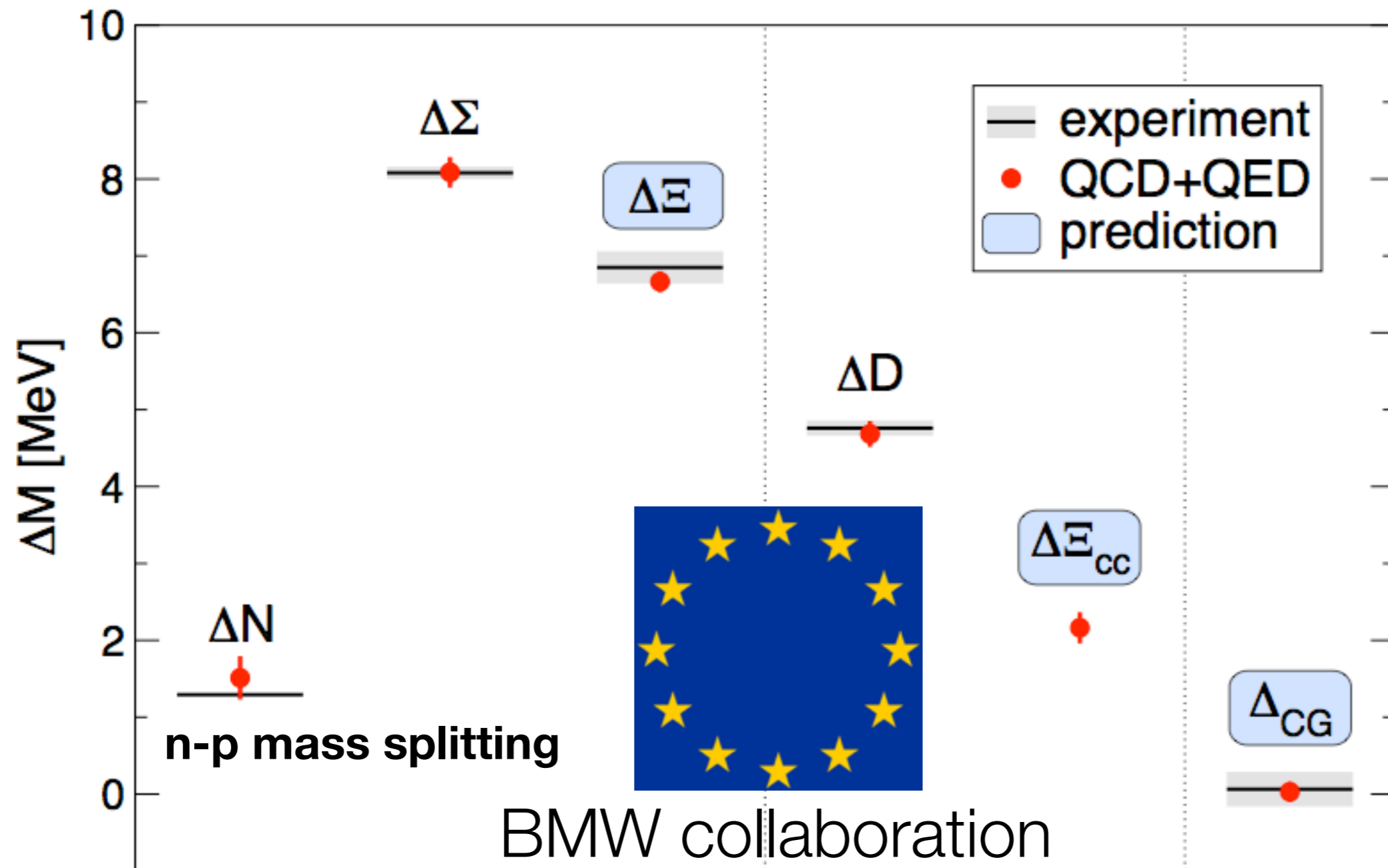
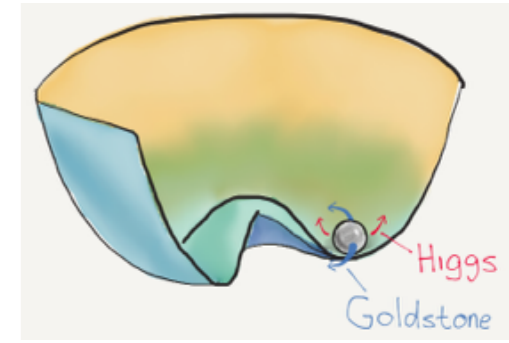


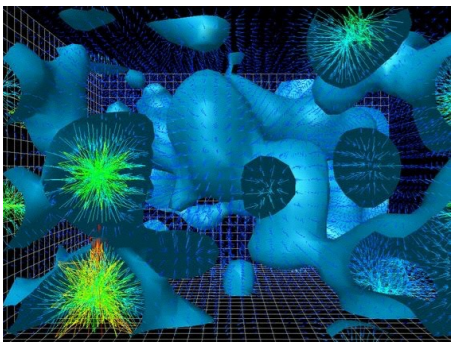
Lattice QCD: Hadron Masses



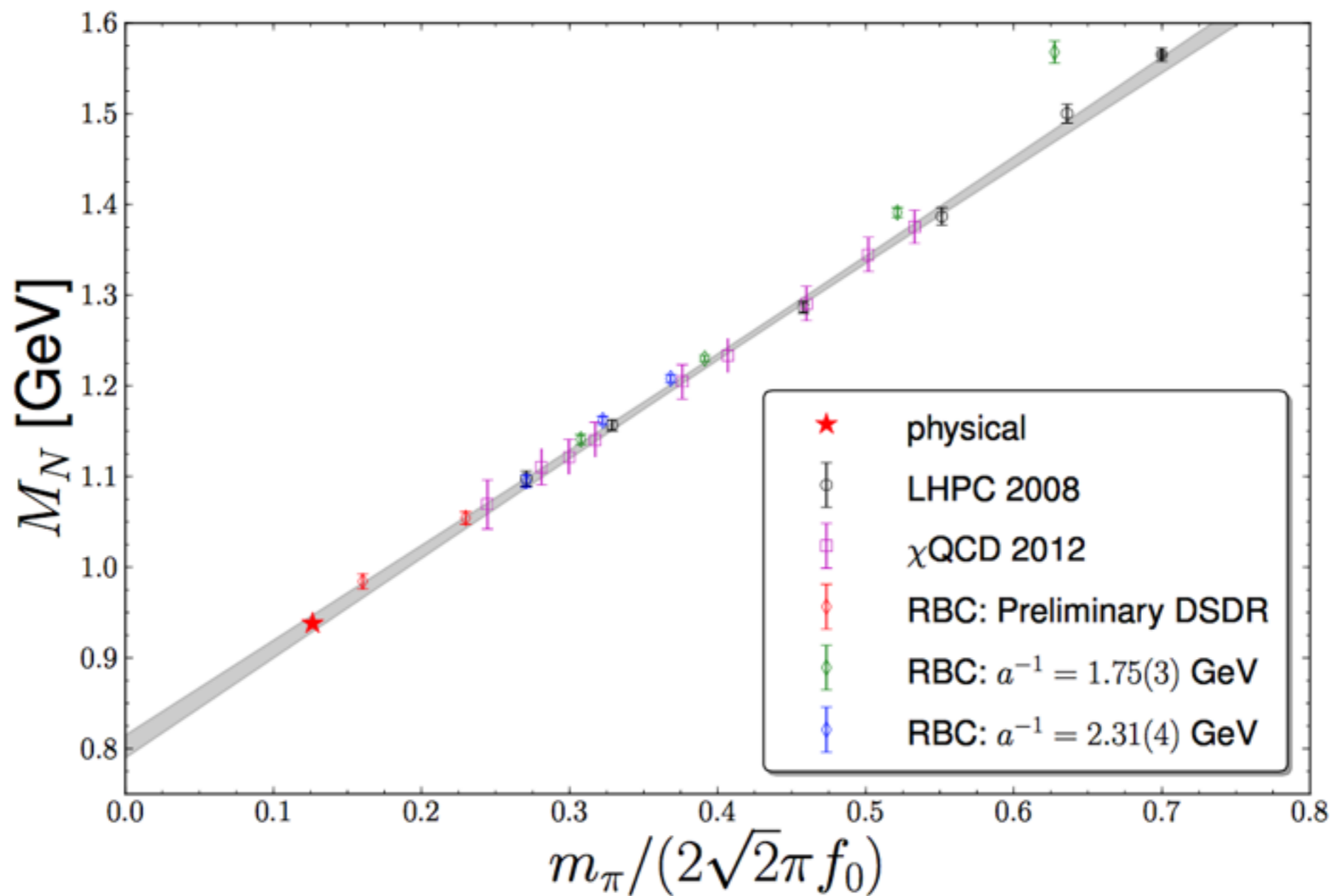
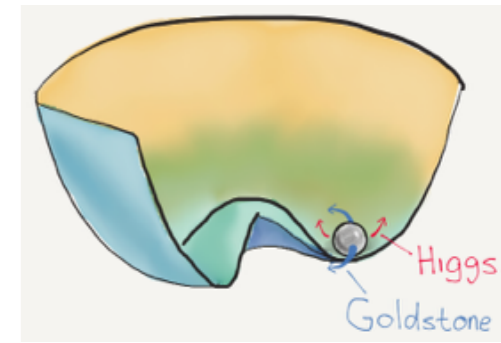


Lattice QCD: The Bleeding Edge

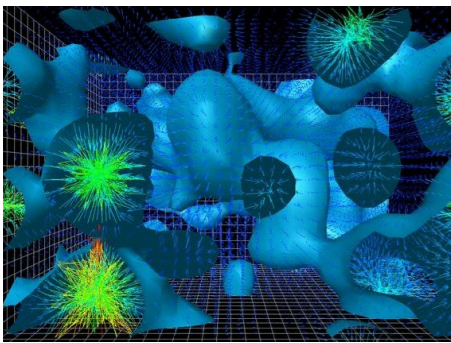




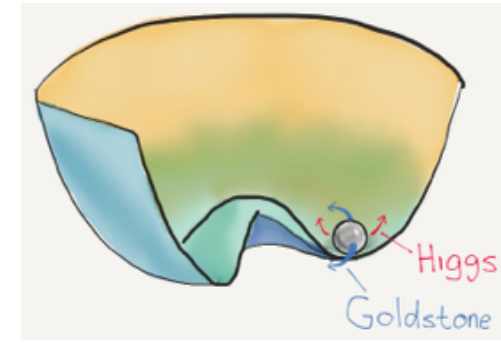
Lattice QCD: Results - Nucleon Mass



$$M_N = 800 \text{ MeV} + m_\pi \quad \text{Unexpected behavior !!}$$



Hadron Mass Decomposition #1



$$\Delta M_{u,d} = M_H(m_\pi) - M_H(0)$$

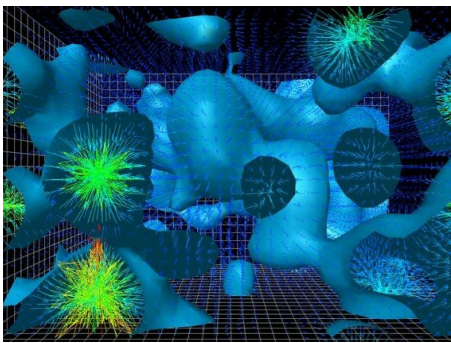
generalizes to any number of the quarks
(NOT the same as turning off the vev)

The difficulty determining the value for vanishing quark mass

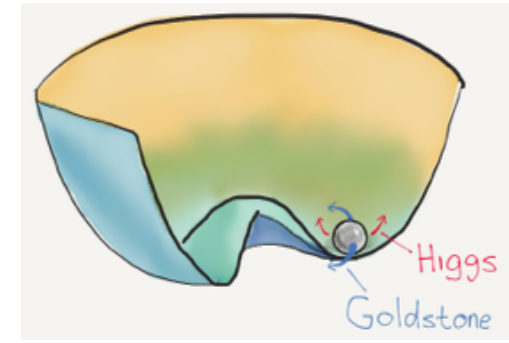
- pion is special - the u,d masses are 100% of its mass
- Nucleon - LQCD and HB χ PT

$$\Delta_m M_N^{n_f=2+1} = M_N(m_u, m_d, m_s)^{n_f=2+1} - M_N^{n_f=2+1}(0, 0, m_s) = 59(4)(7)(3) \text{ MeV}$$

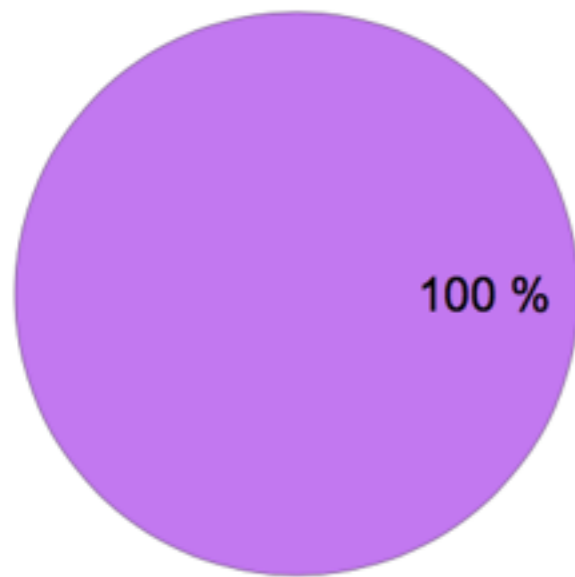
(Alvarez-Ruso et al, 2013,2014)



Hadron Mass Decomposition #1



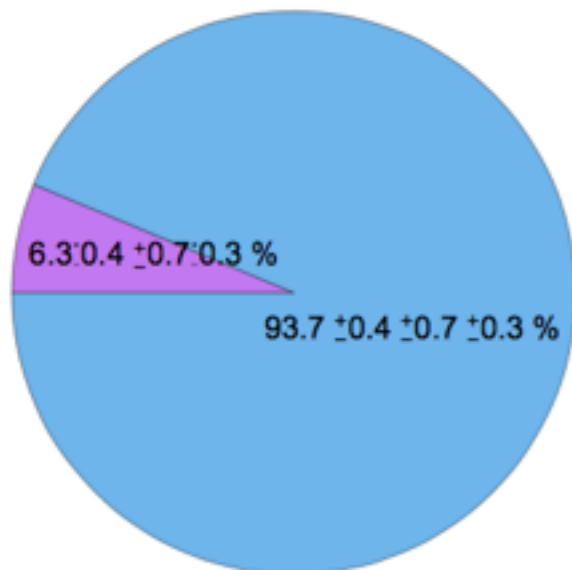
The Pion



- Total U and D Quark Mass Contribution
- U and D Chiral Limit

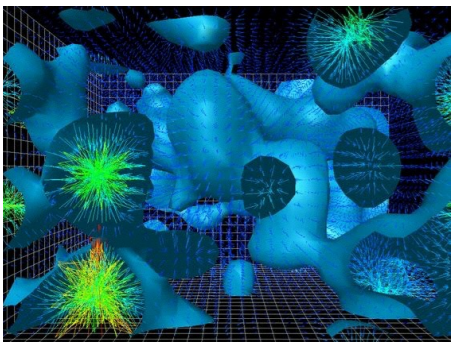
Chiral Symmetry

The Nucleon



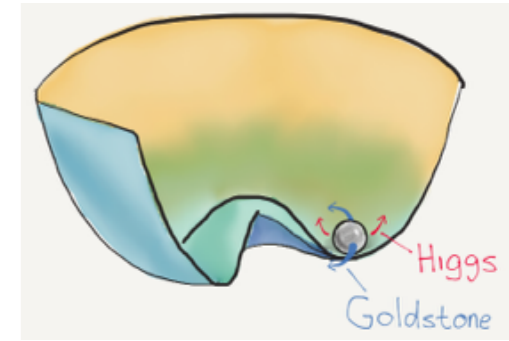
- Total U and D Quark Mass Contribution
- U and D Chiral Limit

LQCD and $HB\chi PT$

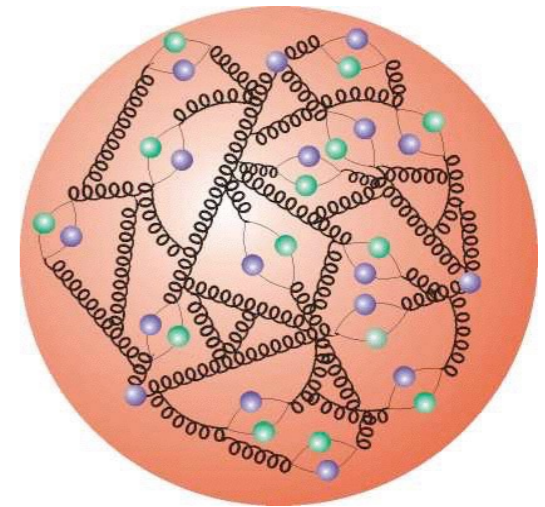


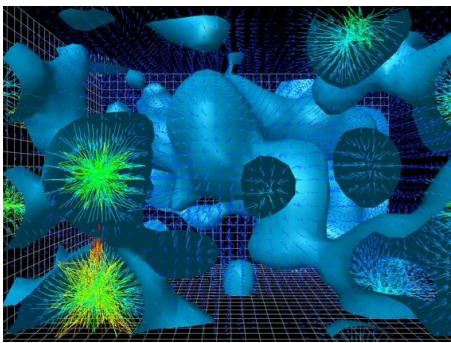
Hadron Mass

Decomposition #1

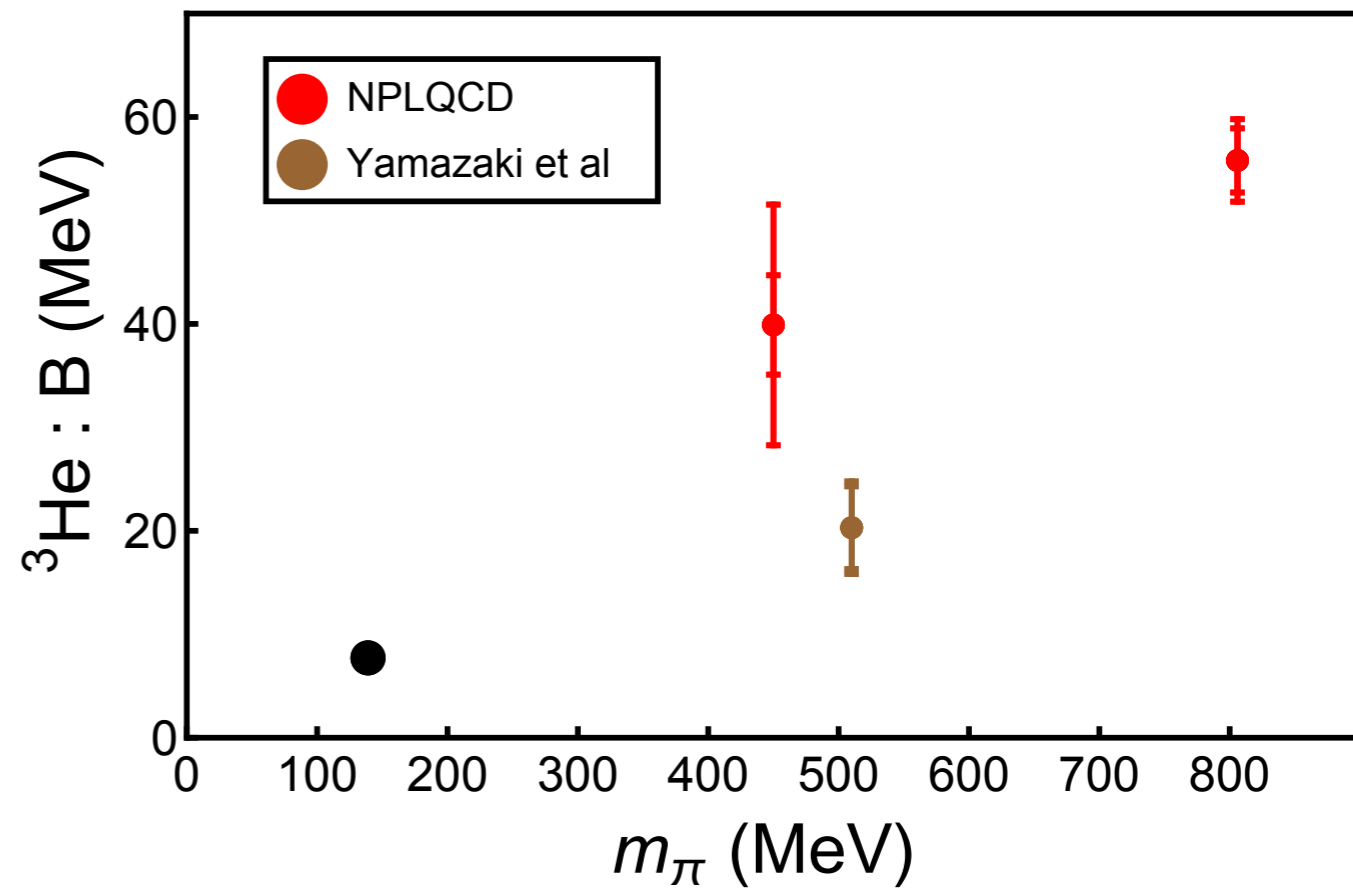
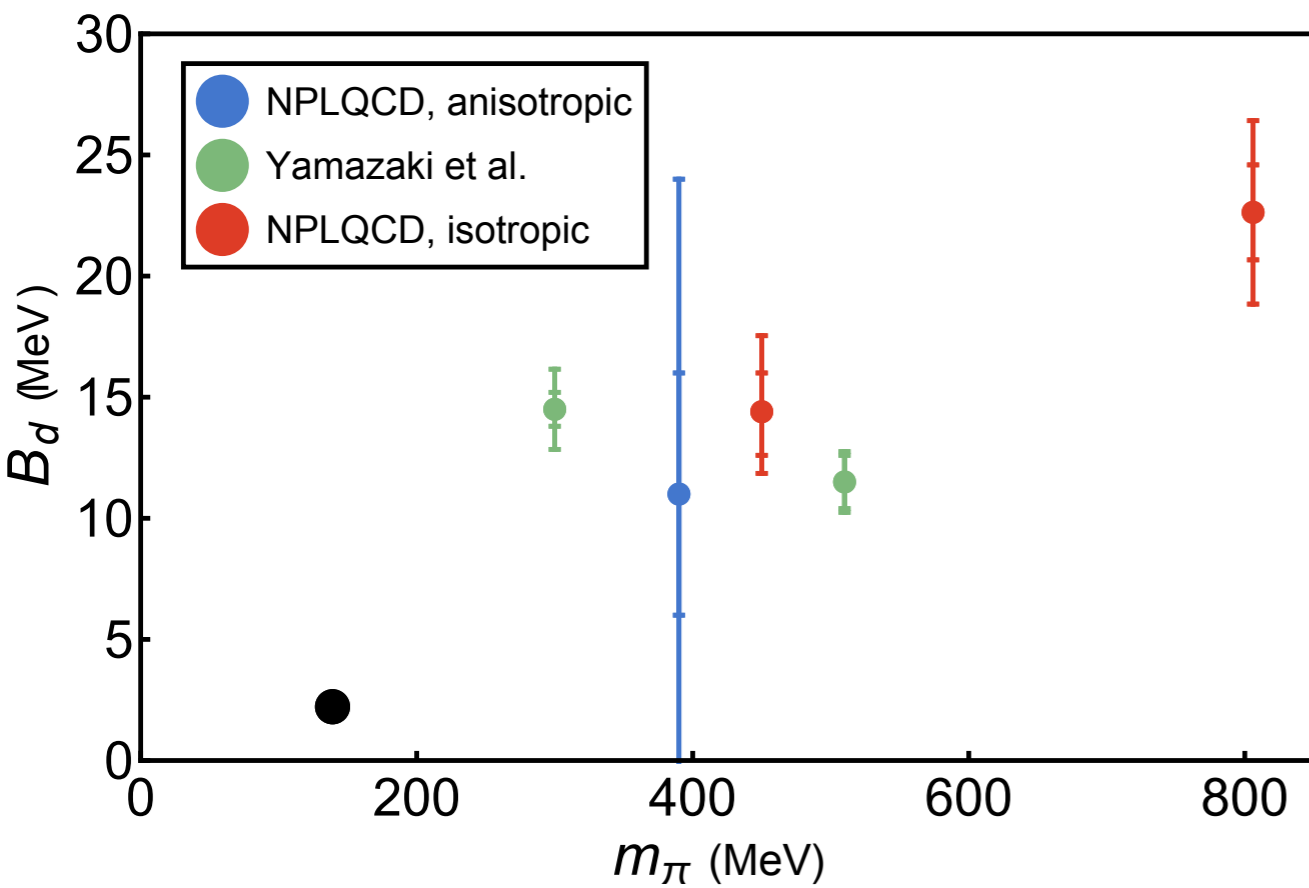
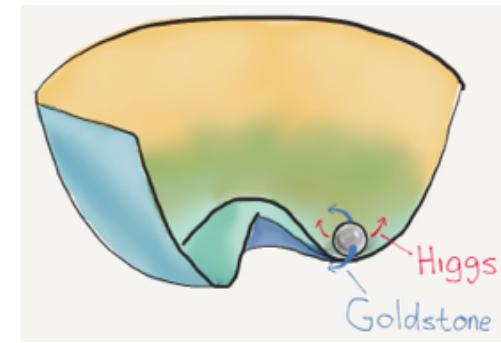


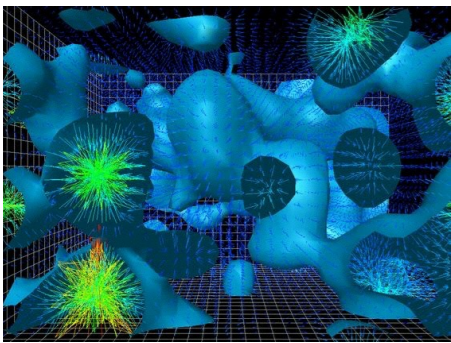
- Accounts for both the *explicit* dependence in the Hamiltonian and *implicit* dependence in the states
- Hadron masses for massless (or near massless) u, d, s , or u, d, s, c , or ... remain to be calculated
 - interesting : higgs vev is lowered to zero
 - scale setting is the issue (in LQCD)
 - fix UV parameters at physical point, then change masses





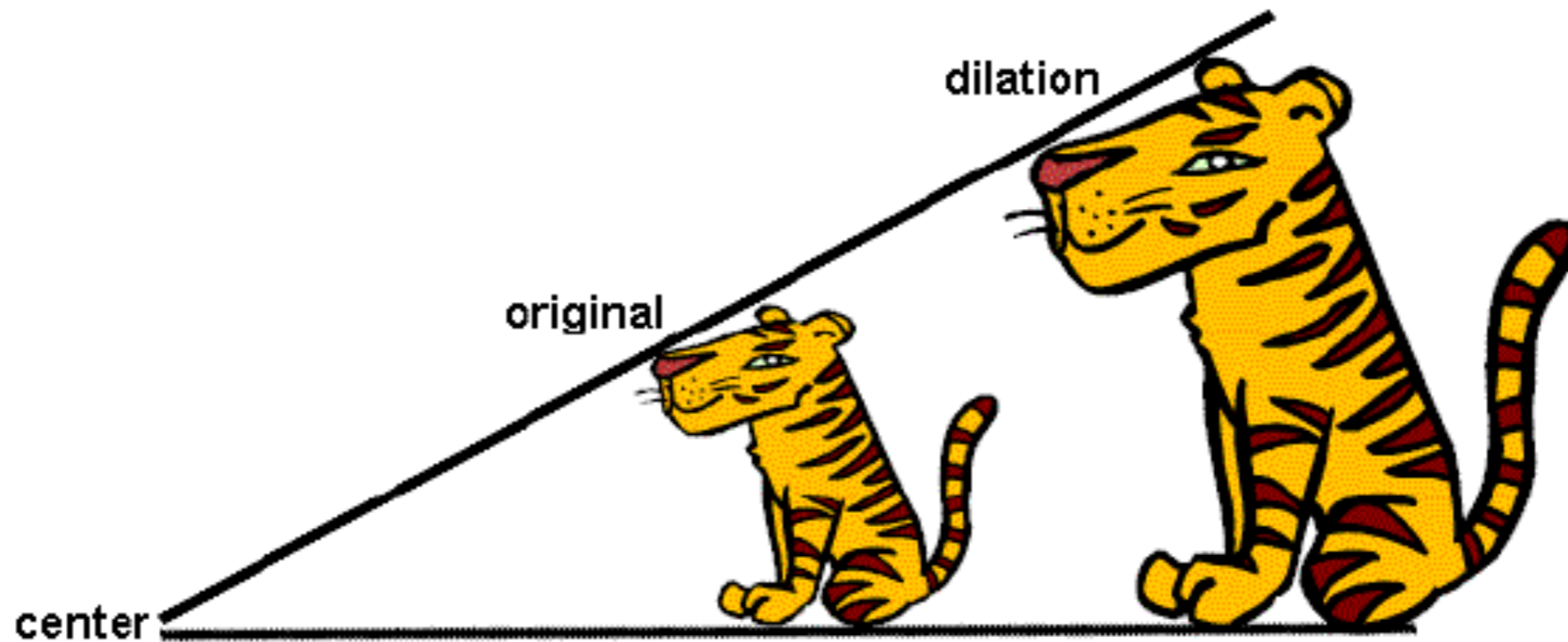
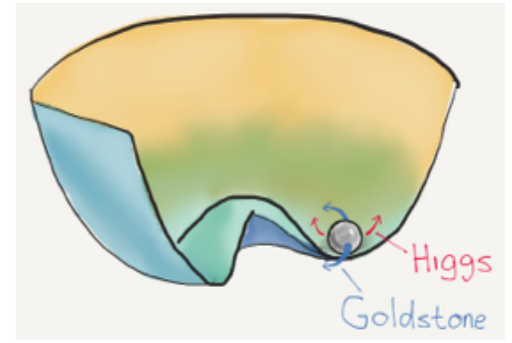
Light Nuclei : Quark Mass Effects





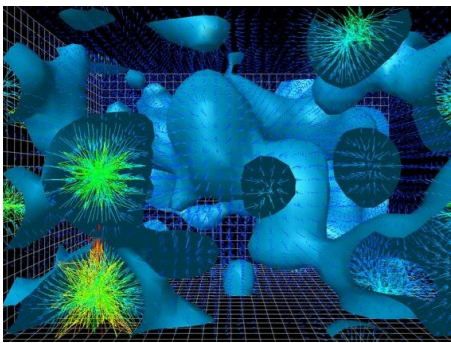
Hadron Mass

The Scale Current



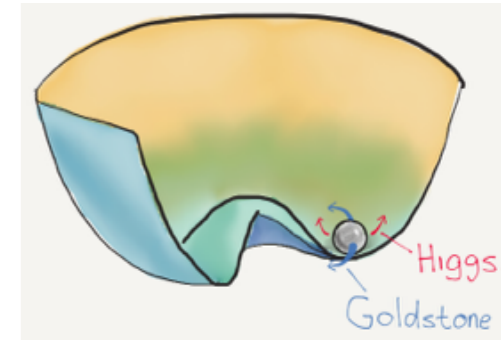
$$s_{\mu} = x^{\nu} T_{\mu\nu}$$

Scale invariance explicitly by masses and quantum fluctuations (through dimensional transmutation)



Hadron Mass

Divergence of the Scale Current



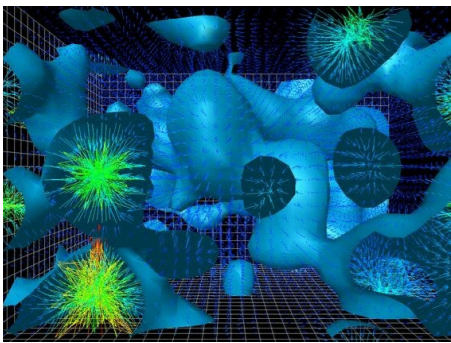
Lorentz Invariant (neglect QED for now)

$$s_\mu = x^\nu T_{\mu\nu}$$

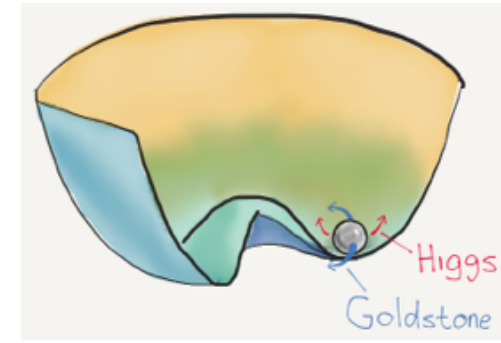
$$\partial^\mu s_\mu = T^\alpha_\alpha = -M_N \bar{N} N \quad \text{Low-Energy EFT}$$

$$\partial^\mu s_\mu = \frac{\beta(\alpha)}{\alpha} \frac{1}{4} G^{\mu\nu} G_{\mu\nu} + (\gamma_m(\alpha) - 1) m_q \bar{q} q \quad \text{QCD}$$

Nucleon Mass comes **entirely** from the *divergence of the scale current*, which is the *trace of the energy-momentum tensor*.



Hadron Mass Decomposition #2



$$M_N = m_q \langle |\bar{q} m_q q| \rangle_{m_q} - m_q \langle | \frac{\beta(\alpha)}{\alpha} \frac{1}{4} G^{\mu\nu} G_{\mu\nu} + \gamma_m(\alpha) m_q \bar{q} q | \rangle_{m_q}$$

$$= M_{\text{ExM}} + M_{\text{ExA}} = \sigma + M_{\text{ExA}}$$

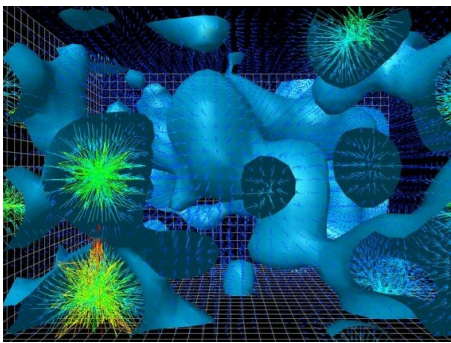


Derivative at the physical point

LQCD (χ QCD collaboration results)

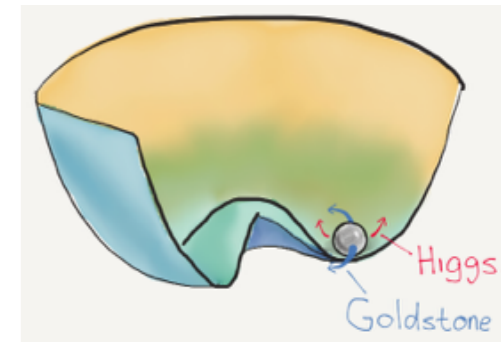
$$\sigma_{ud}^{n_f=2+1} = 44.4(3.2)(4.5)\text{MeV}, \quad \sigma_s^{n_f=2+1} = 32.3(4.7)(4.8)\text{MeV}$$

$$\sigma_{uds}^{n_f=2+1} = 76.7(6.4)(6.6)\text{MeV}$$

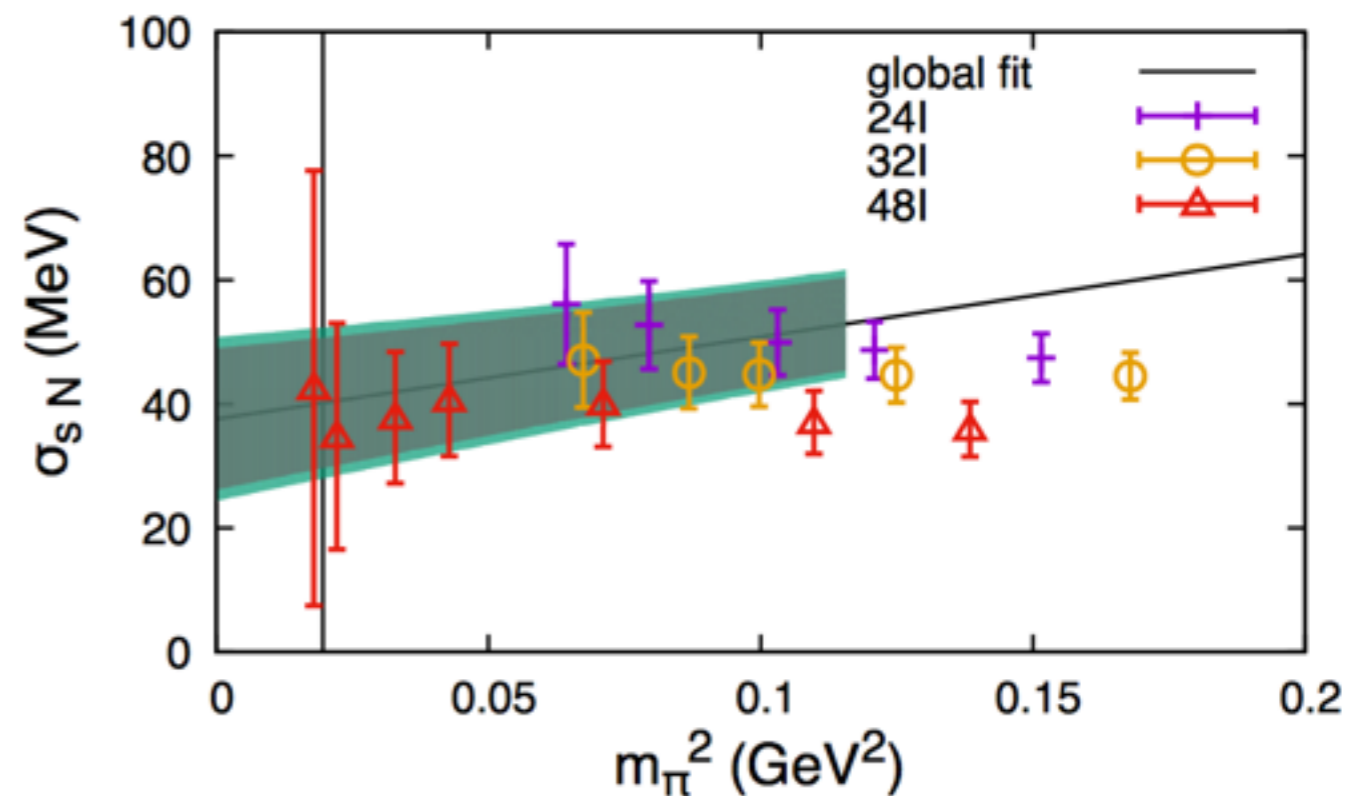
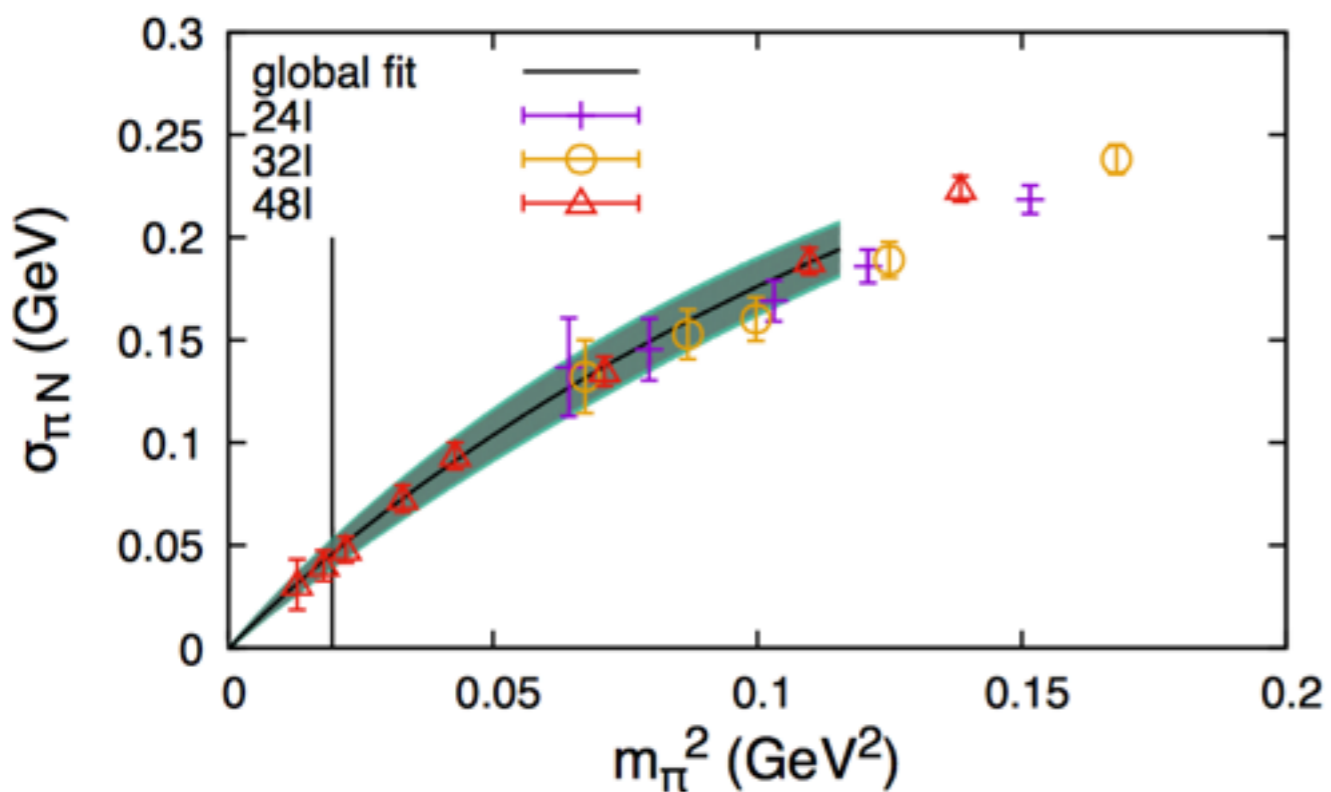


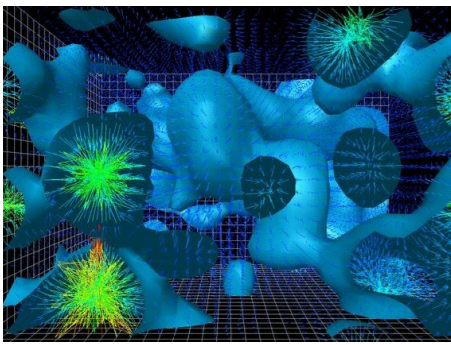
Lattice QCD: (ChiQCD)

Results - Nucleon σ -Term



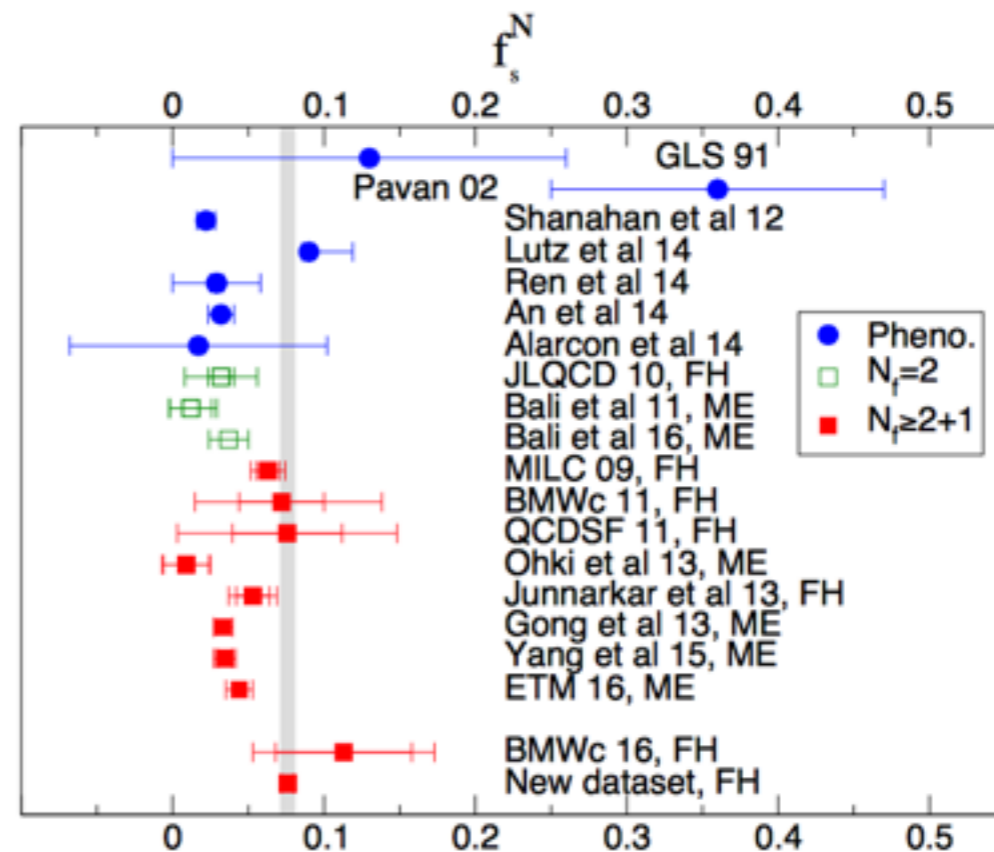
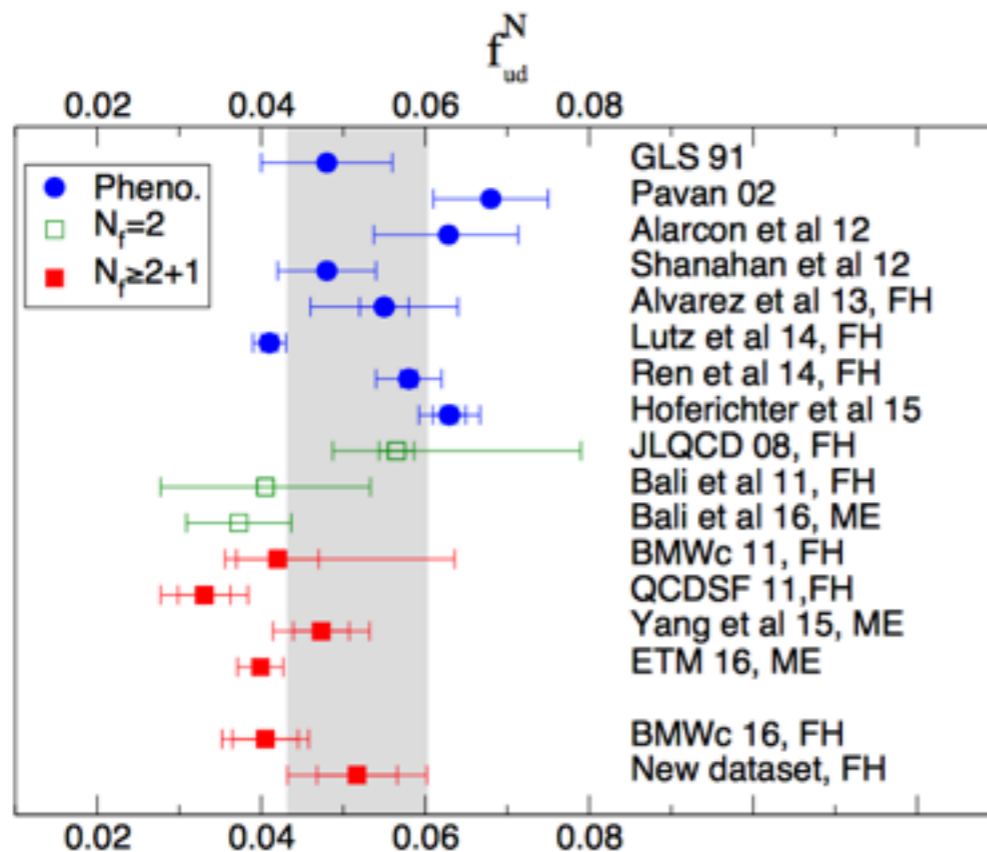
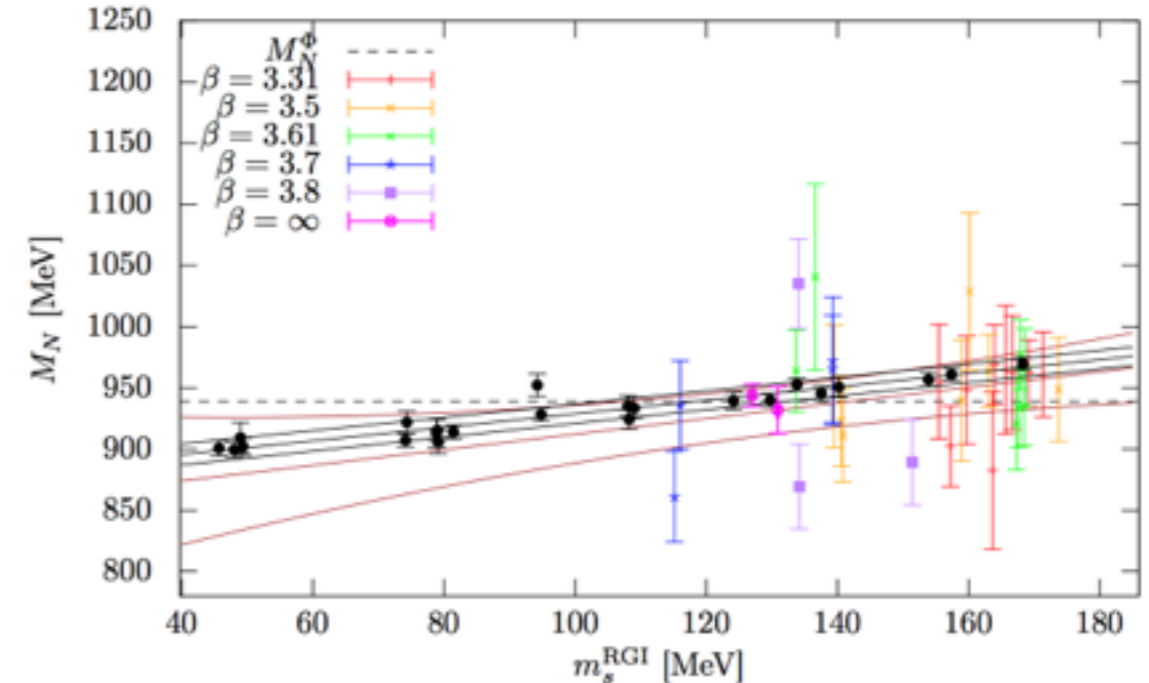
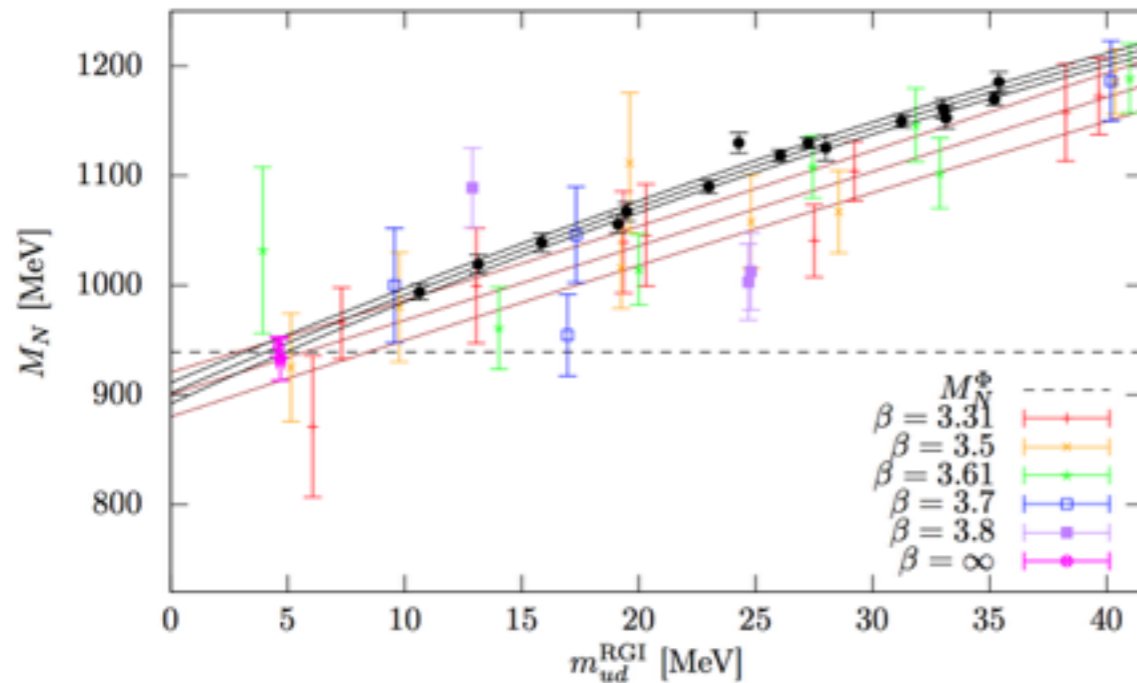
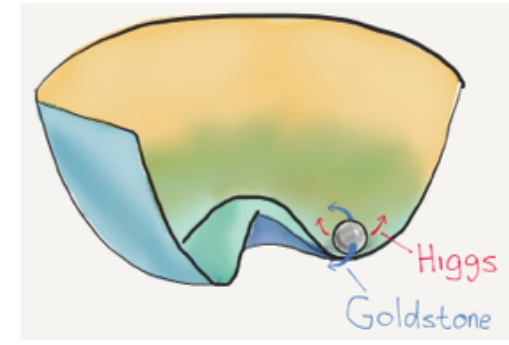
Recent results of chiral fermion calculations

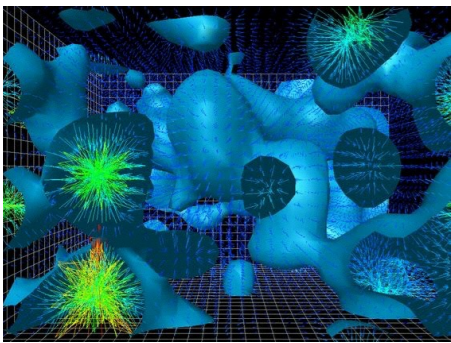




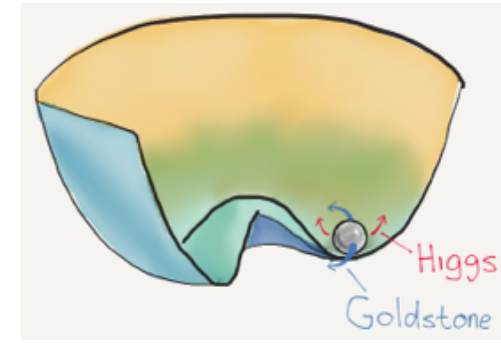
Lattice QCD: (BMW)

Results - Nucleon σ -Term





Hadron Mass Decomposition #2



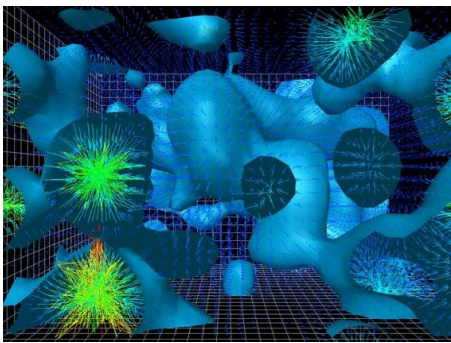
$$\begin{aligned}
 M_N &= m_q \langle |\bar{q} m_q q| \rangle_{m_q} - m_q \langle | \frac{\beta(\alpha)}{\alpha} \frac{1}{4} G^{\mu\nu} G_{\mu\nu} + \gamma_m(\alpha) m_q \bar{q} q | \rangle_{m_q} \\
 &= M_{\text{ExM}} + M_{\text{ExA}} = \sigma + M_{\text{ExA}}
 \end{aligned}$$

The Pion is special : chiral symmetry
Gell-Mann-Oakes-Renner

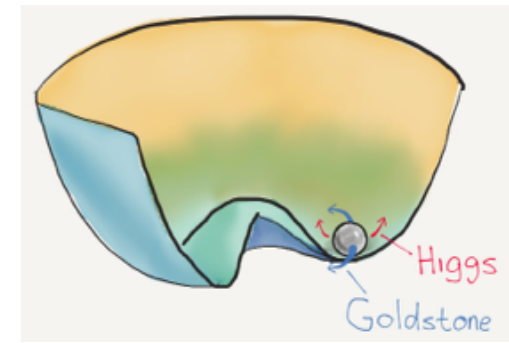
$$m_\pi = \sqrt{\mu(m_u + m_d)}$$

$$\frac{\partial m_\pi}{\partial m_u} = \frac{1}{2} \sqrt{\frac{\mu}{m_u + m_d}}$$

$$\sum_{q=u,d} m_q \frac{\partial m_\pi}{\partial m_q} = \frac{1}{2} \mu \sqrt{m_u + m_d} = \frac{1}{2} m_\pi$$

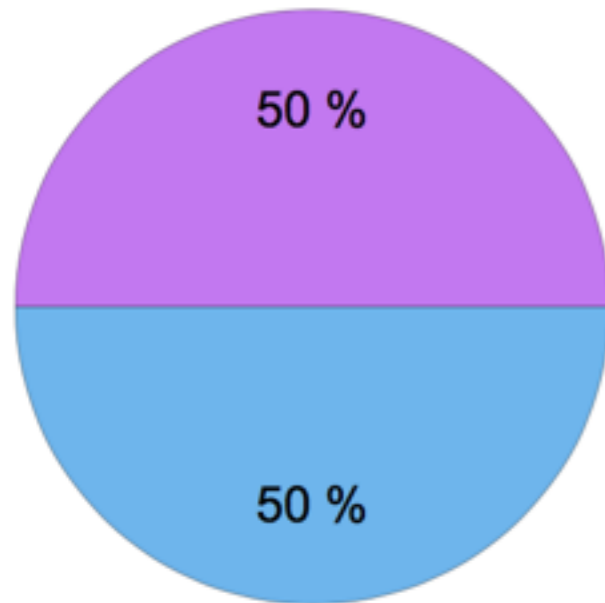


Hadron Mass Decomposition #2



u- and d-quarks only

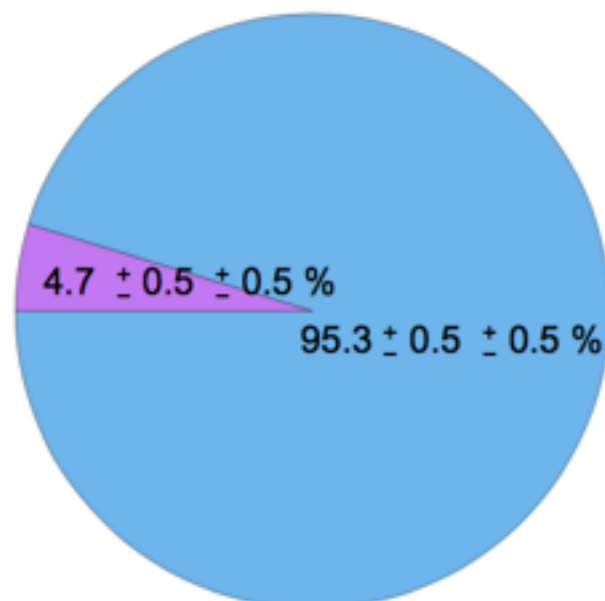
The Pion



- Explicit U and D Quark-Mass Contributions: σ_{ud} -Term
- Explicit Anomalous Contribution

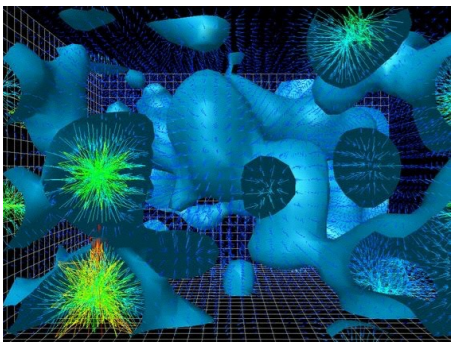
GMOR

The Nucleon

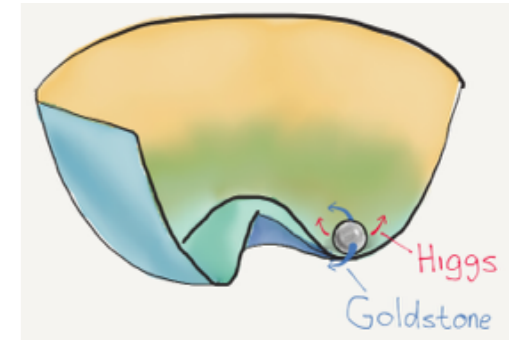


- Explicit U and D Quark-Mass Contributions: σ_{ud} -Term
- Explicit Anomalous Contribution

LQCD

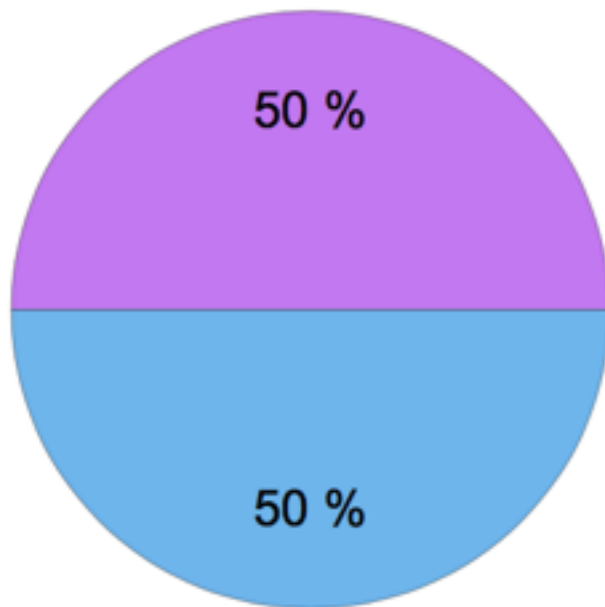


Hadron Mass Decomposition #2



u-, d- and s-quarks only

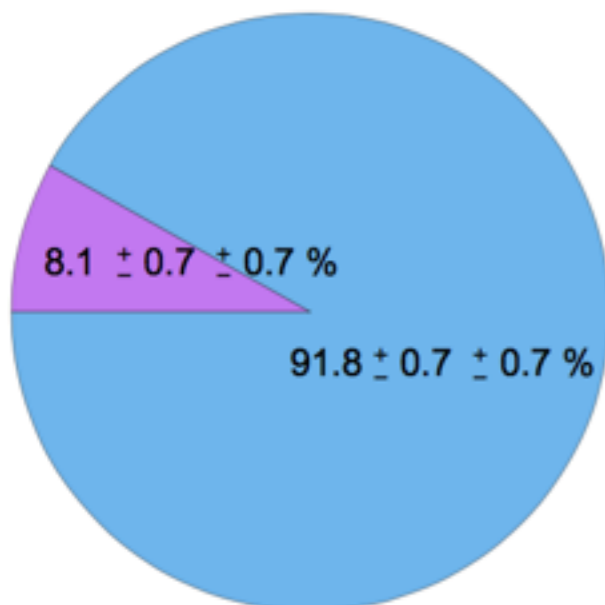
The Pion



- Explicit U, D, and S Quark-Mass Contributions: σ_{uds} -Term
- Explicit Anomalous Contribution

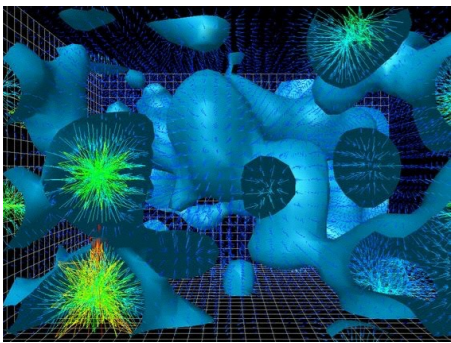
GMOR

The Nucleon

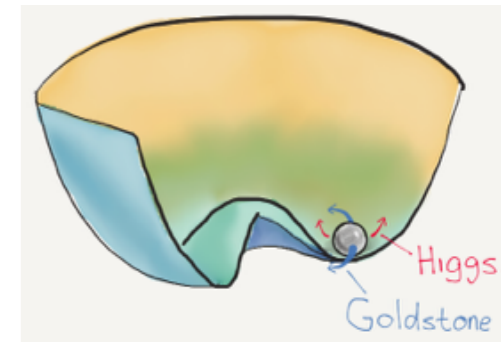


- Explicit U, D, and S Quark-Mass Contributions: σ_{uds} -Term
- Explicit Anomalous Contribution

LQCD



Nuclear σ -Terms and Dark Matter Interactions



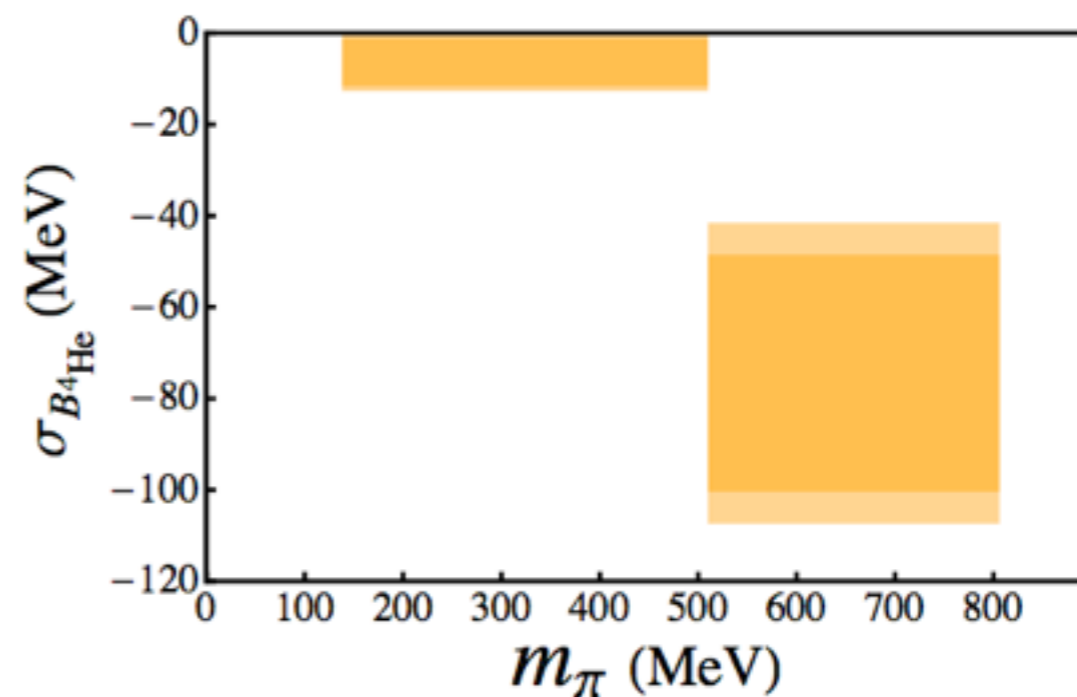
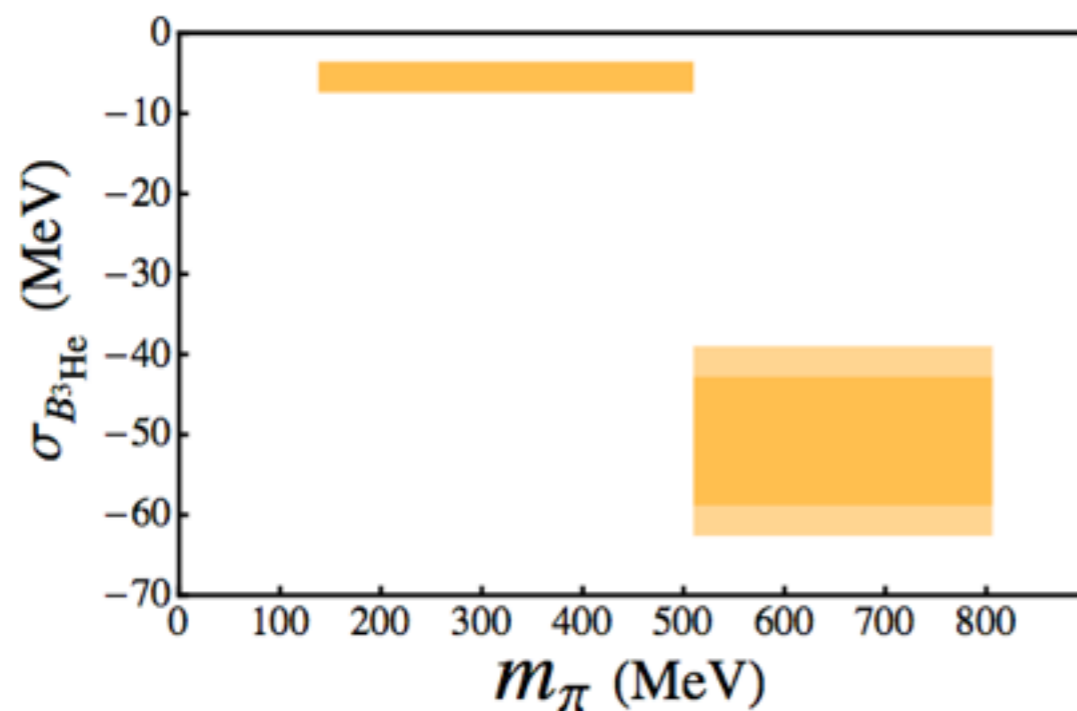
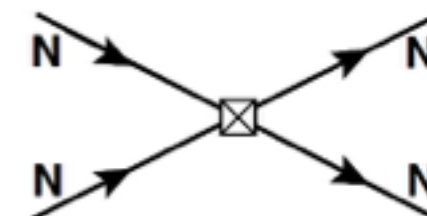
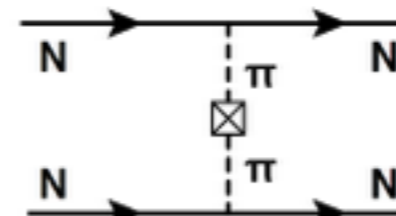
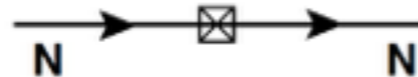
Nuclear σ -terms

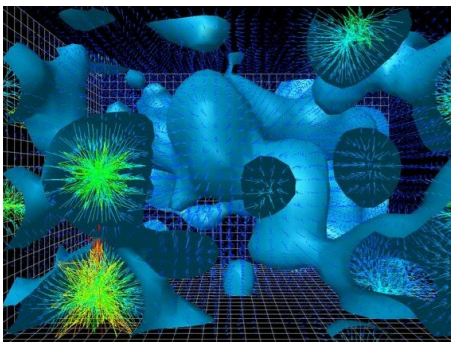
(Beane *et al*, Phys.Rev. D89 (2014) 074505)

$$\begin{aligned}\sigma_{Z,N} &= \bar{m} \langle Z, N(\text{gs}) | \bar{u}u + \bar{d}d | Z, N(\text{gs}) \rangle = \bar{m} \frac{d}{d\bar{m}} E_{Z,N}^{(\text{gs})} \\ &= \left[1 + \mathcal{O}(m_\pi^2) \right] \frac{m_\pi}{2} \frac{d}{dm_\pi} E_{Z,N}^{(\text{gs})}\end{aligned}$$

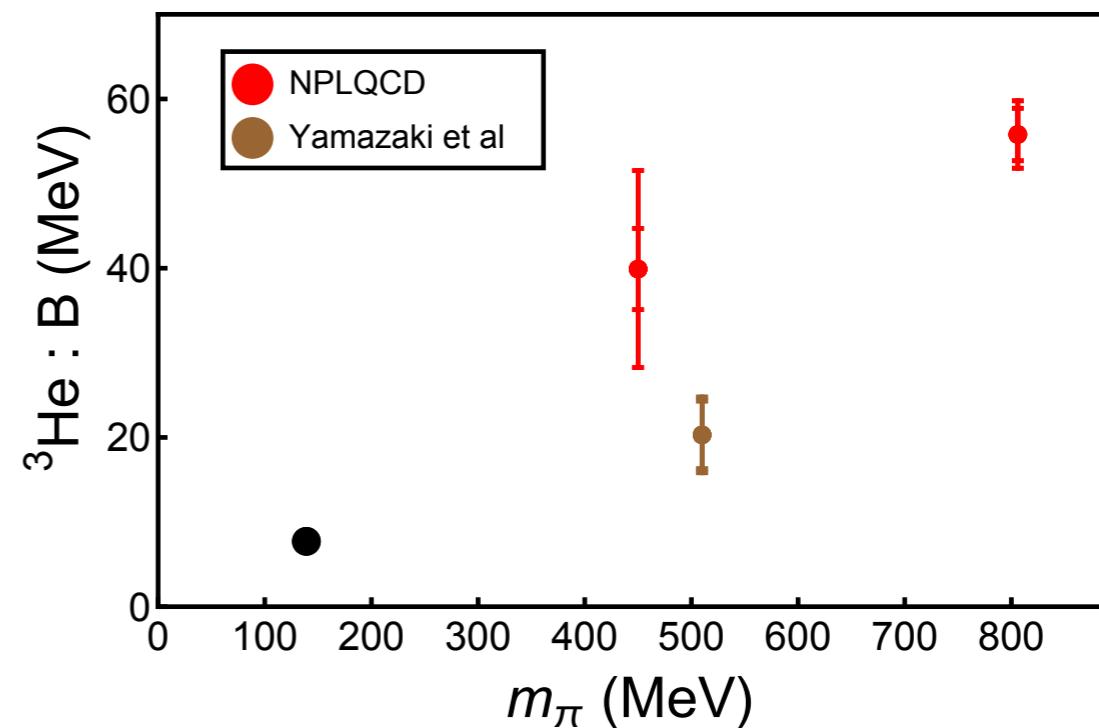
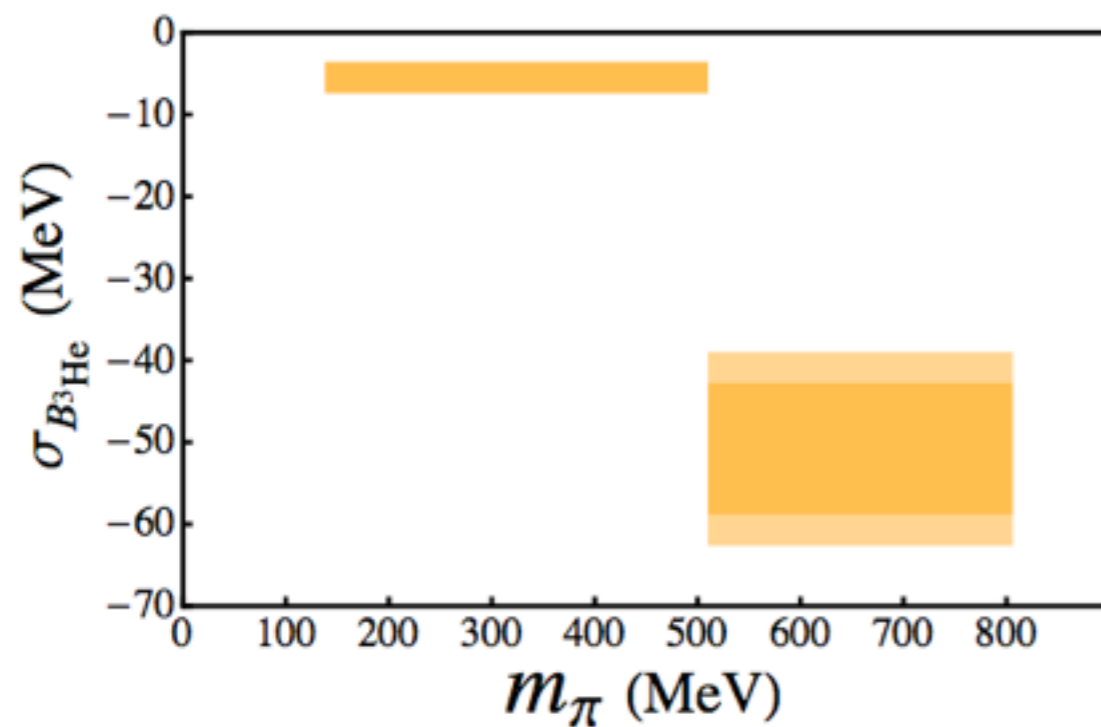
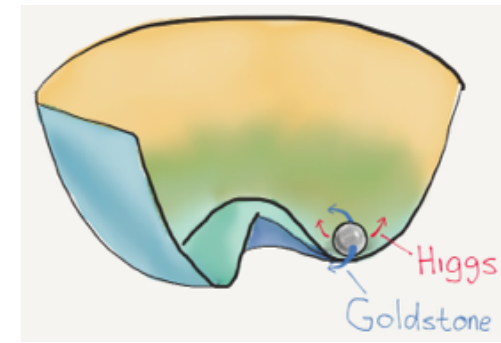


σ -term from
binding only





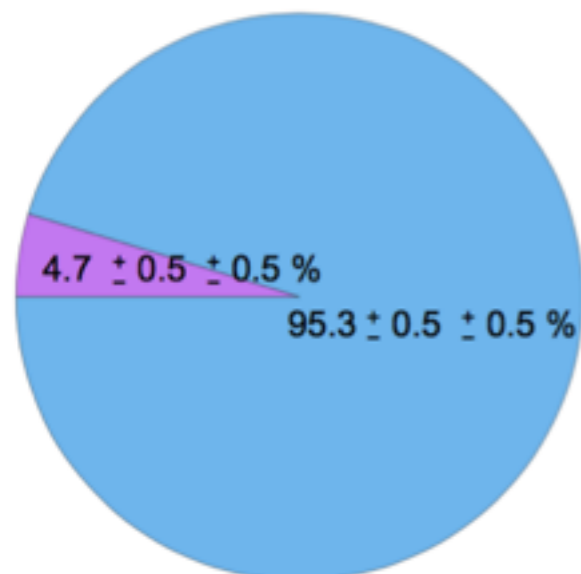
Nuclear σ -Terms and Dark Matter Interactions



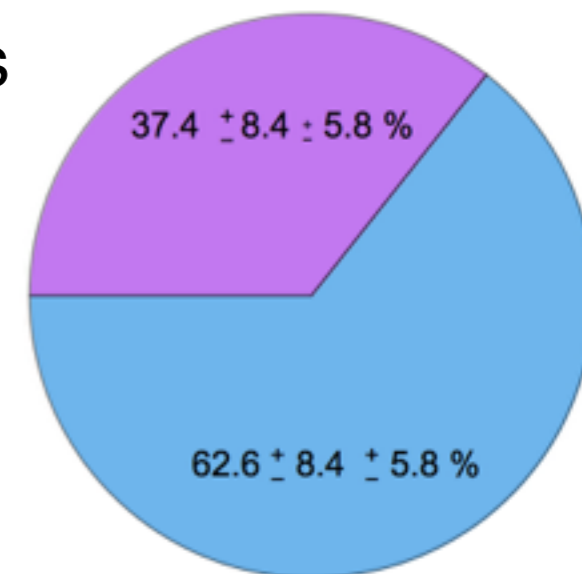
Nucleon Mass

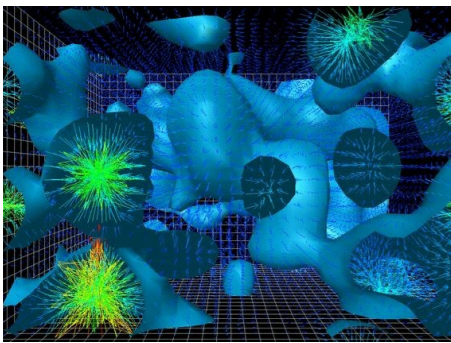
Binding ^3He ($m_\pi=450$ MeV)

Dictates a Class of Dark Matter Interactions

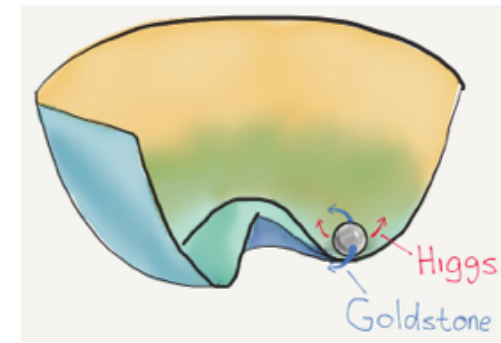


- Explicit U and D Quark-Mass Contributions: σ_{ud} -Term
- Explicit Anomalous Contribution





Hadron Mass Decomposition #3



The Rest Frame

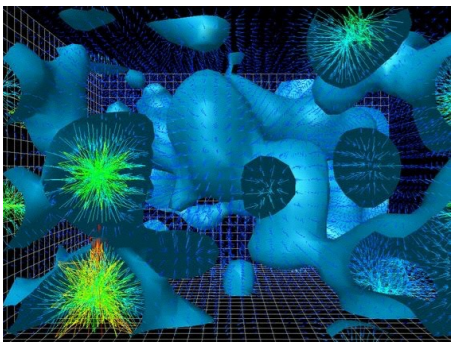
$$H_m = \sum_q \bar{q} m_q q = H_{\text{ExM}}$$

$$H_{\text{kin}} = -\frac{1}{2} (|\mathbf{E}|^2 - |\mathbf{B}|^2) + \sum_q \bar{q} \boldsymbol{\gamma} \cdot \mathbf{D} q = H_{\text{ExYM+Qkin}}$$

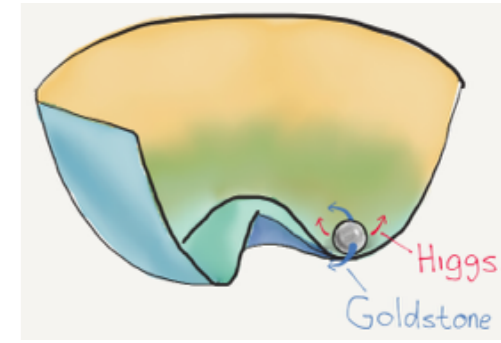
$$H_a = -\frac{1}{4} \left[\gamma_m \sum_q m_q \bar{q} q + \frac{\beta}{4\alpha} (|\mathbf{E}|^2 + |\mathbf{B}|^2) \right] = \frac{1}{4} H_{\text{ExA}}$$

$$M_N = -m_q \langle |T^{00}| \rangle_{m_q} = m_q \langle |H_m| \rangle_{m_q} + m_q \langle |H_{\text{kin}}| \rangle_{m_q} + m_q \langle |H_a| \rangle_{m_q}$$

$$m_q \langle |H_{\text{ExYM+Qkin}}| \rangle_{m_q} = \frac{3}{4} m_q \langle |H_{\text{ExA}}| \rangle_{m_q} = 3 m_q \langle |H_a| \rangle_{m_q}$$



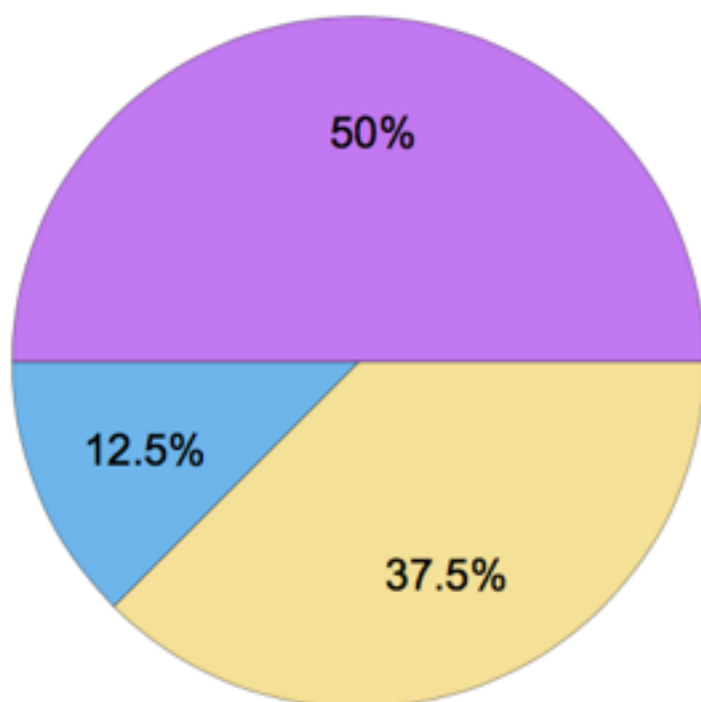
Hadron Mass Decomposition #3



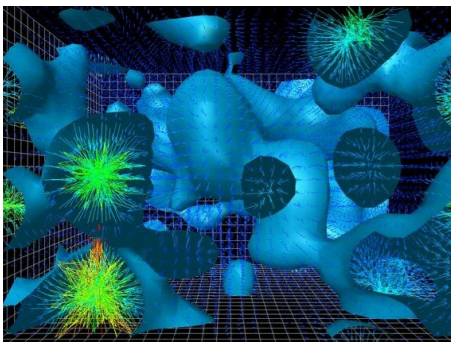
$$m_{\pi} \sim \left(\frac{1}{2} m_{\pi} \right)_{\text{ExM}} + \left(\frac{3}{8} m_{\pi} \right)_{\text{ExYM+Qkin}} + \left(\frac{1}{8} m_{\pi} \right)_{\text{ExA}}$$

The Pion

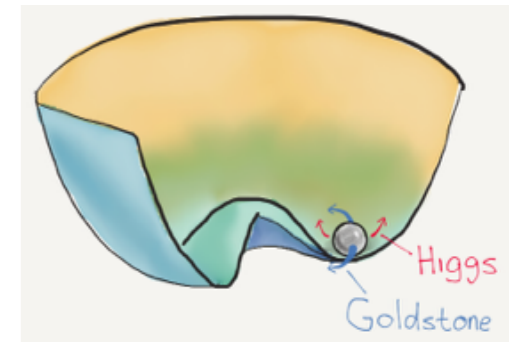
GMOR



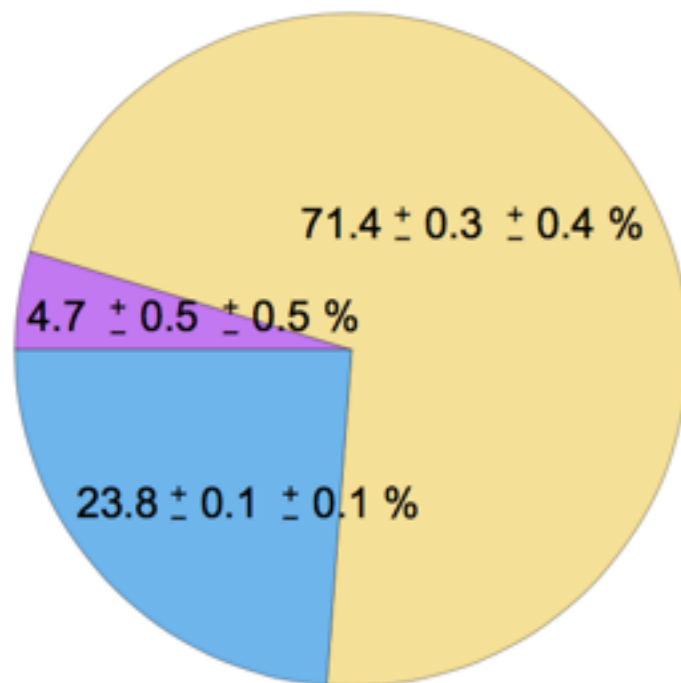
- Explicit U and D Quark-Mass Contributions: σ_{ud} -Term
- Explicit Yang-Mills and Quark Kinetic Contributions
- Explicit Anomalous Contribution



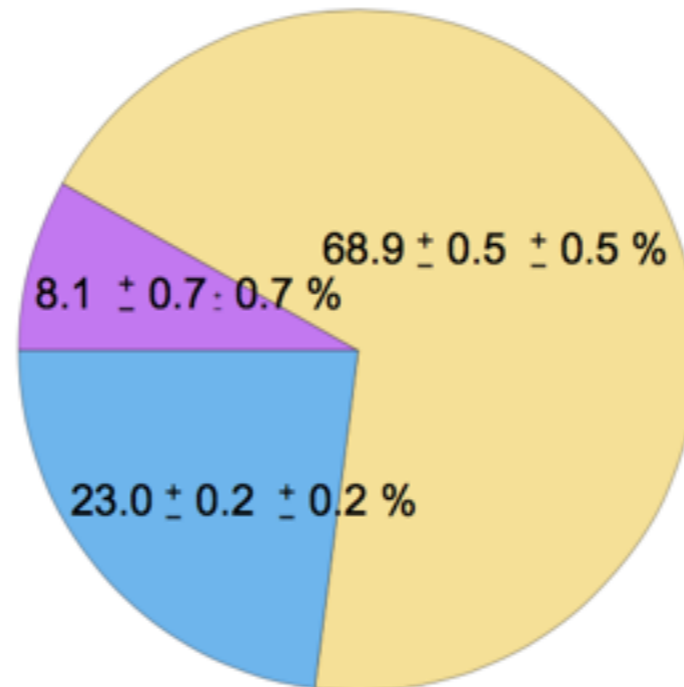
Hadron Mass Decomposition #3



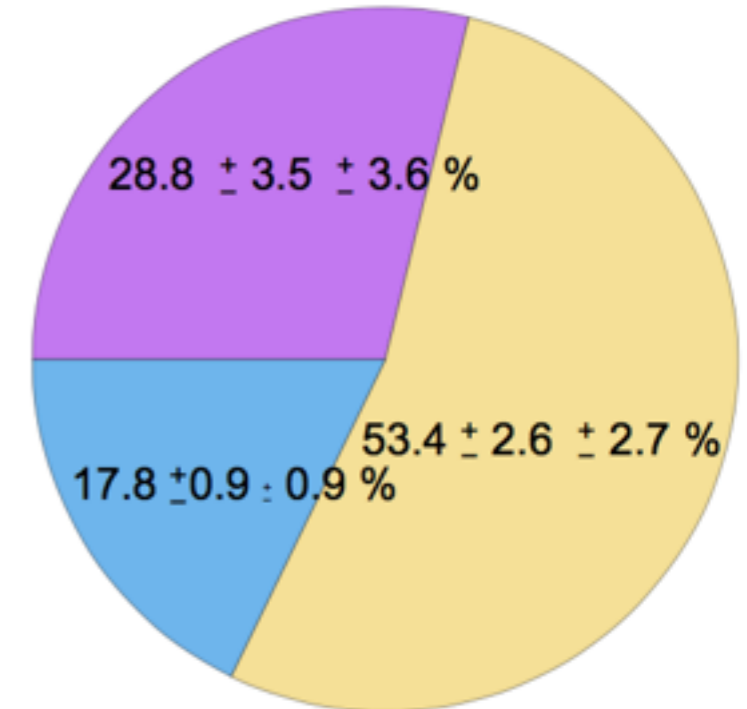
Nucleon Mass



u,d



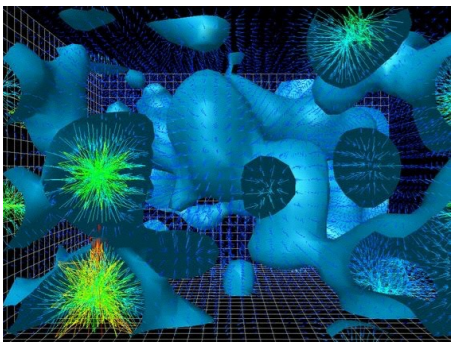
u,d,s



u,d,s,c,b,t

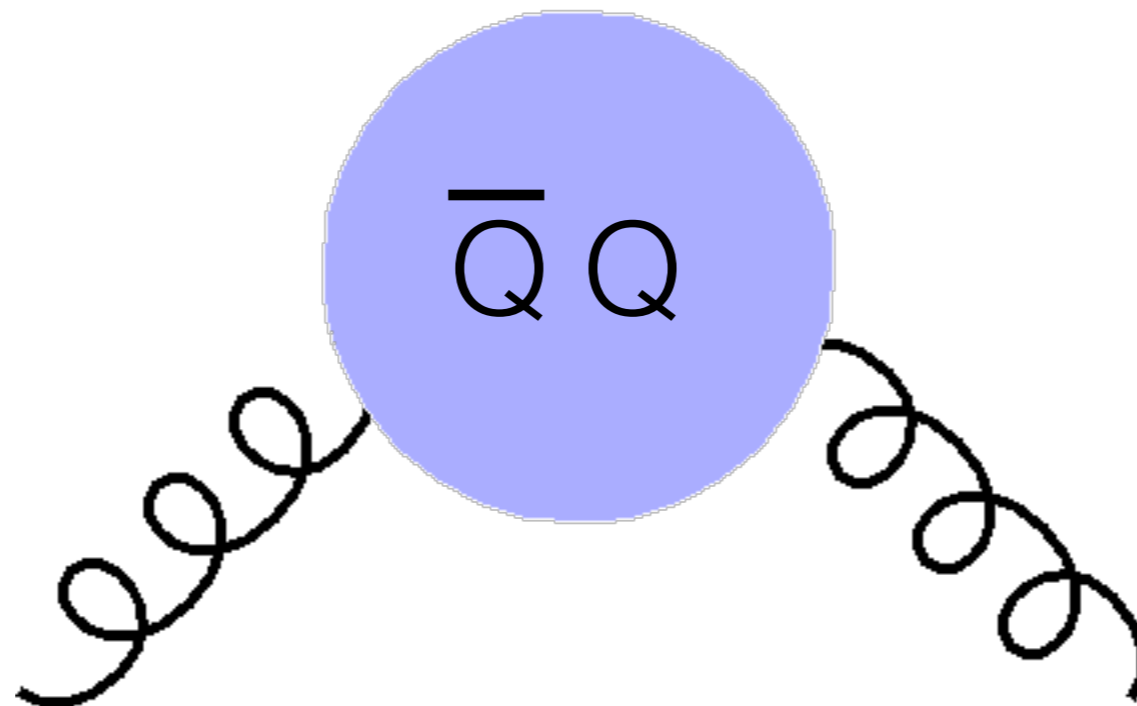
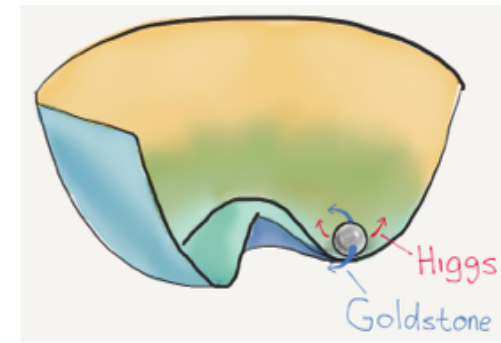
- Explicit U and D Quark-Mass Contributions: σ_{ud} -Term
- Explicit Yang-Mills and Quark Kinetic Contributions
- Explicit Anomalous Contribution

pQCD extrapolations for c,b,t



Experiment

Quarkonia Interactions



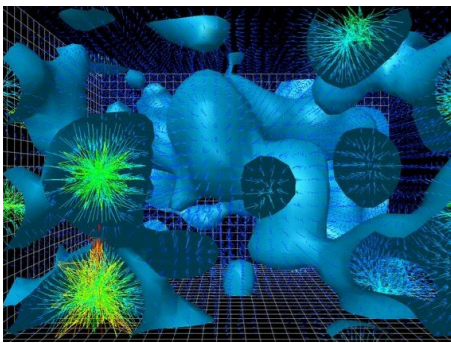
Only spin-0 and spin-2 , no spin-1 (requires 3 gluons)

$$G_{\mu\nu} G^{\mu\nu}, G_{0\alpha} G_0^\alpha$$

$$O(1) \quad (\text{Luke, Manohar, MJS})$$

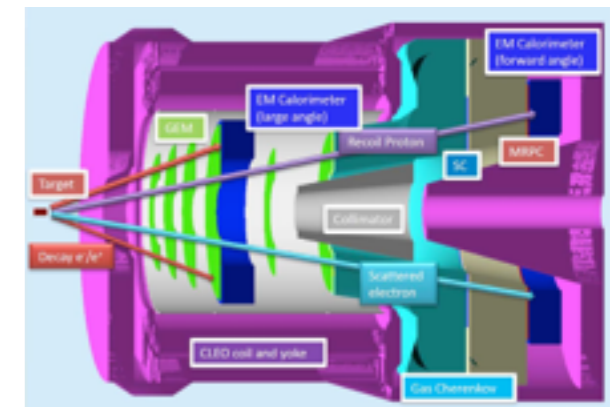
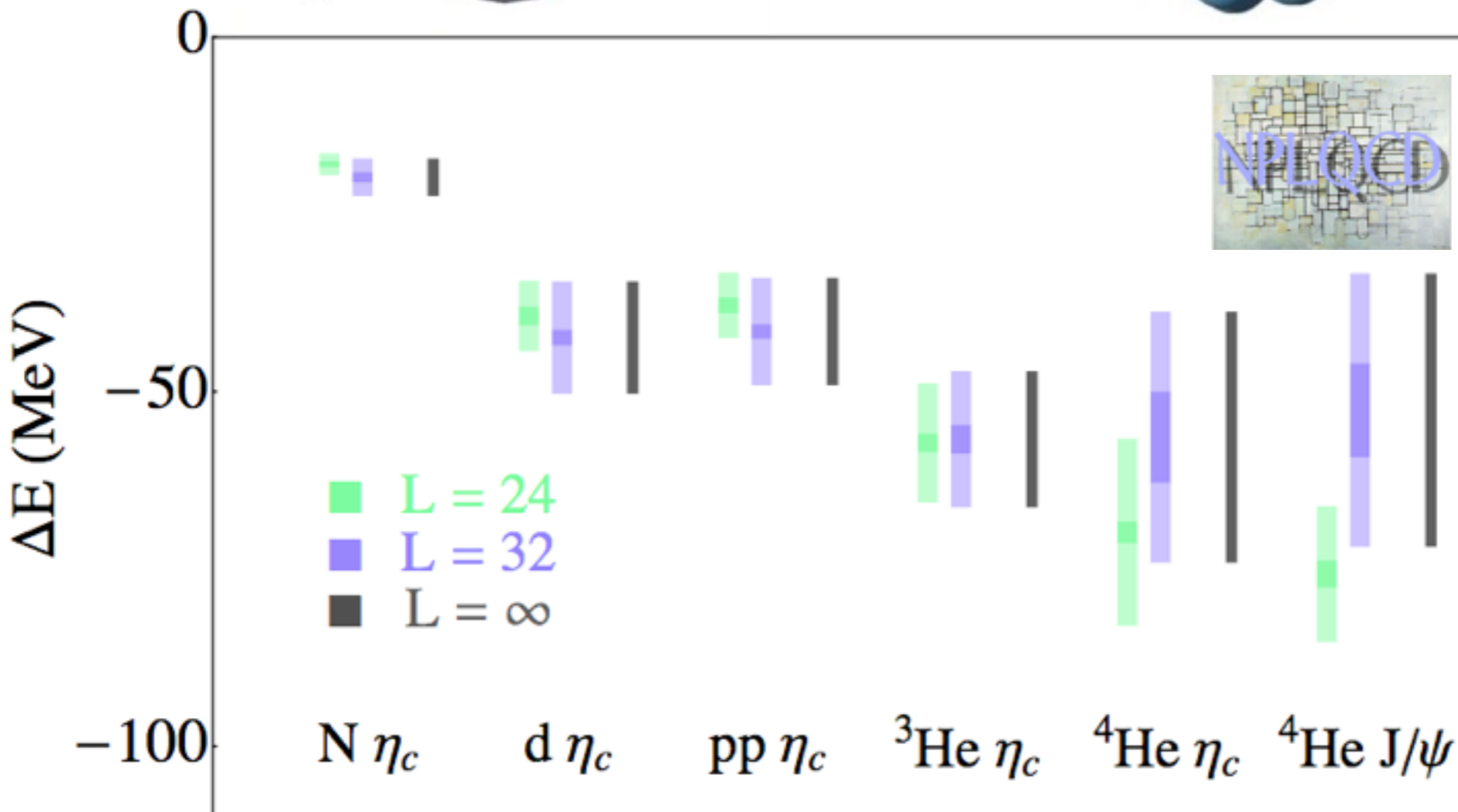
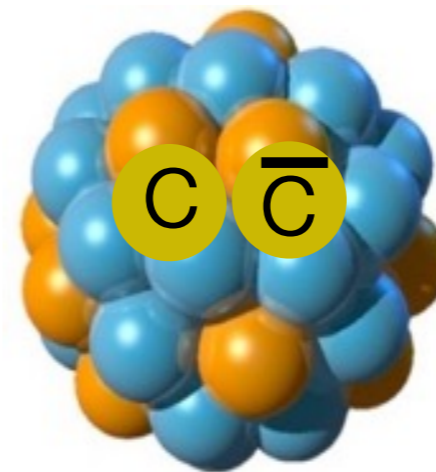
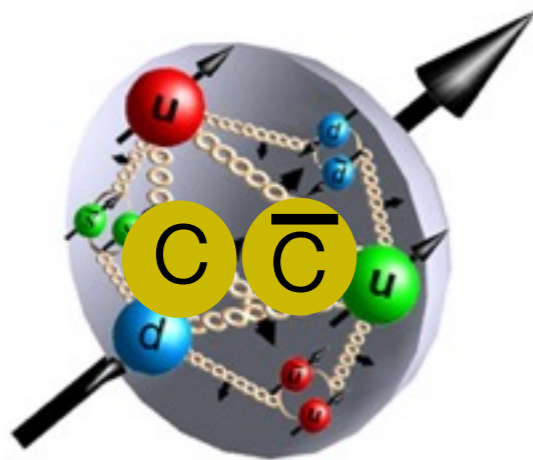
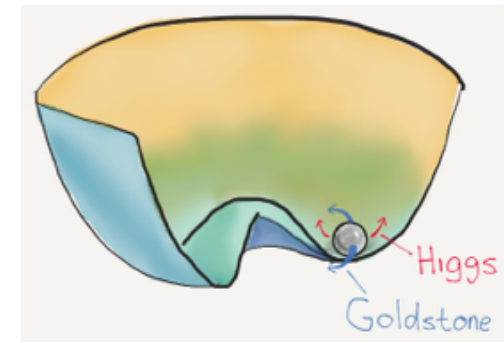
$$G_{i\alpha} G_j^\alpha \text{ symmetrized, traceless}$$

$$O(1/M_Q)$$



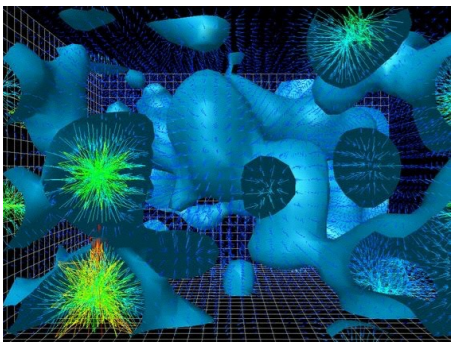
Charmonium-Nuclei

$$m_{\pi} \sim 800 \text{ MeV}$$

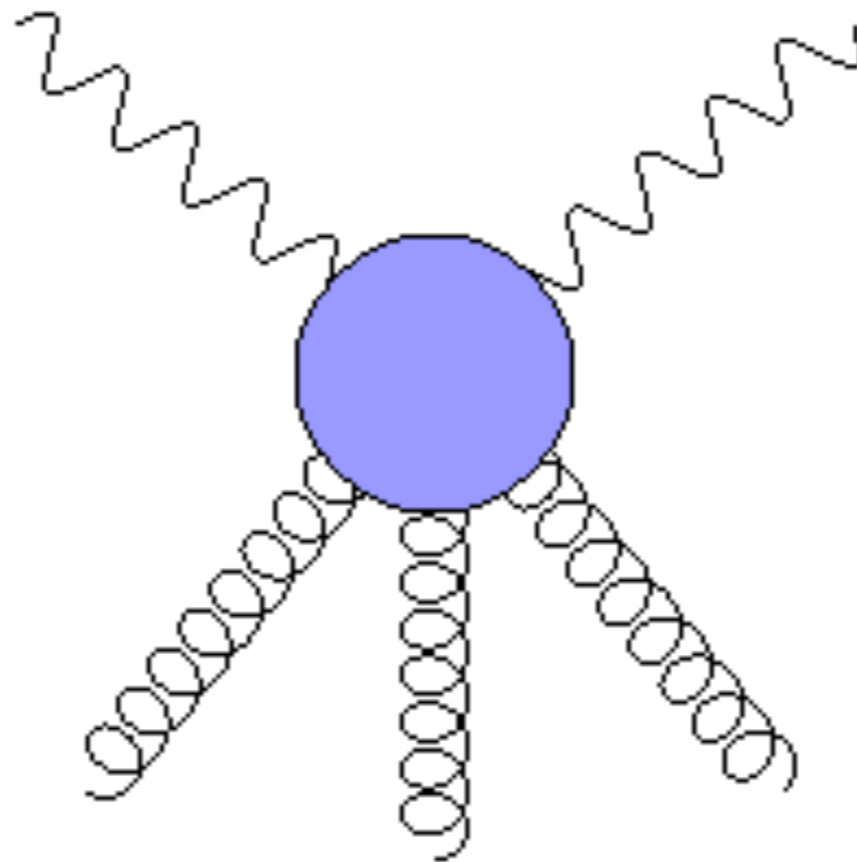
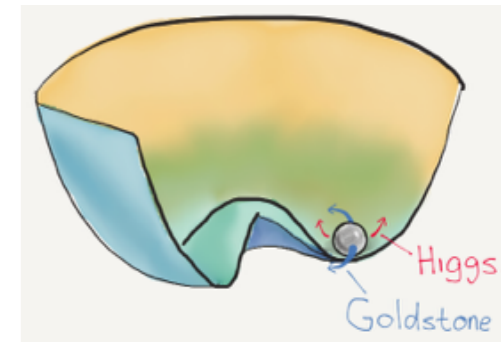


Athenna

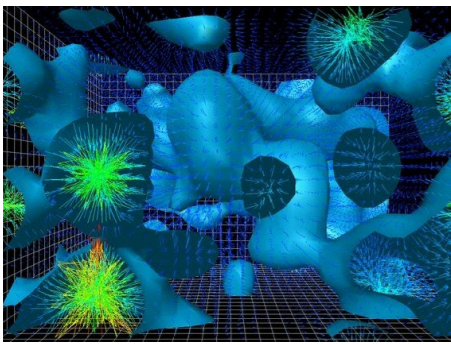




Experiment Other Probes?

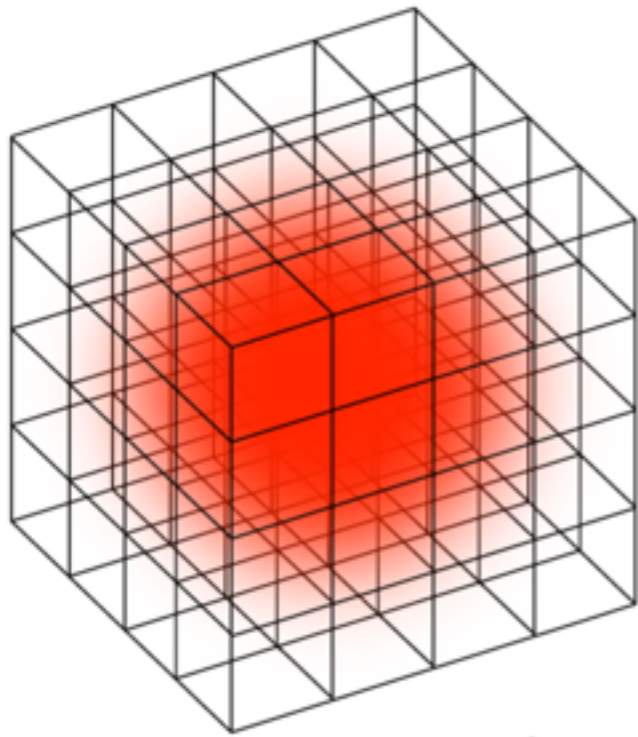
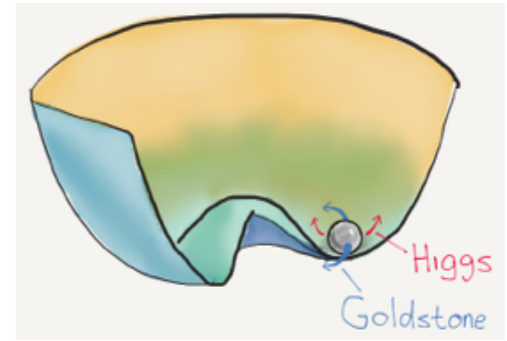


- Can other experimentally accessible probes be identified ?
- Momentum transfer ?
- Probes related to T_{μ}^{ν} are interesting



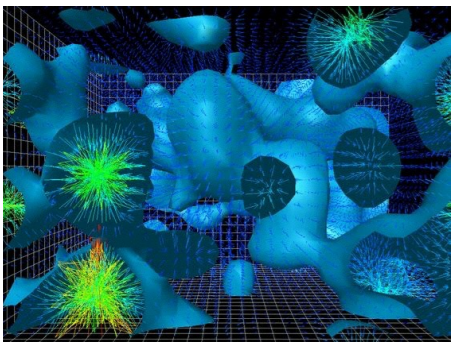
Hadron Mass

Chiral Limit from LQCD?



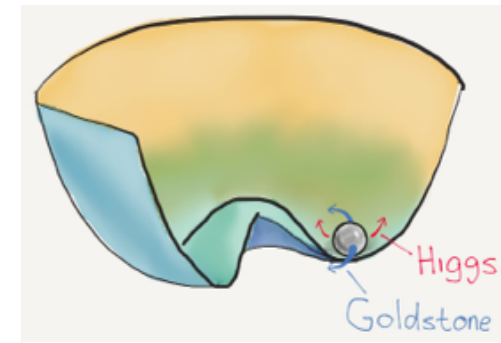
Mass of the nucleon when $v_{ev}=0$?

- 1) Tune the UV parameters to experiment at the physical quark masses
- 2) Reduce the quark masses with fixed UV parameters
- 3) Can do u, d, s, c, b but not t

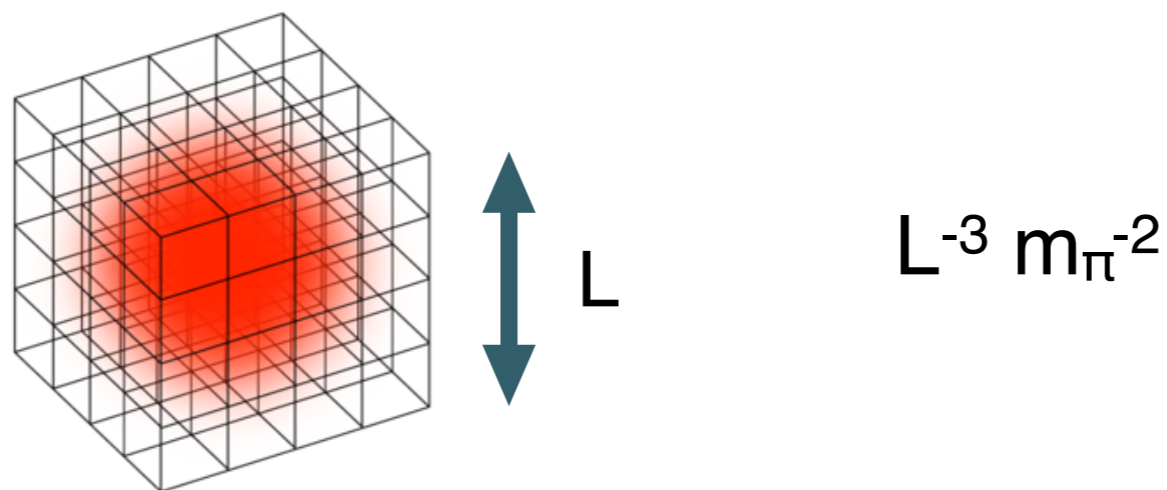


Hadron Mass

Chiral Limit from LQCD?



Interesting question - but challenging ...

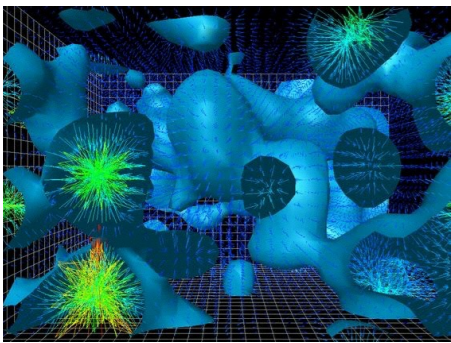


Resum zero modes in path integral $L m_{\pi} \ll 2 \pi$

Becomes unclear!

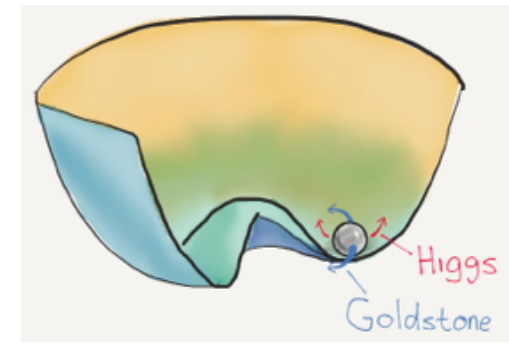
Want lattice volumes such that $L m_{\pi} > 2 \pi$

Interesting calculations to do

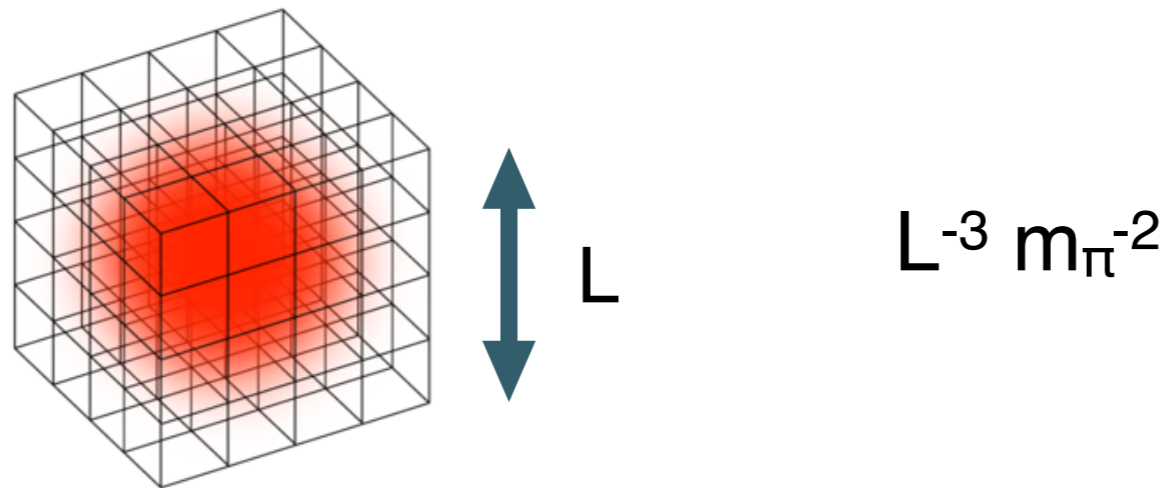


Nuclei

Chiral Limit from LQCD?



Interesting question - but challenging ...

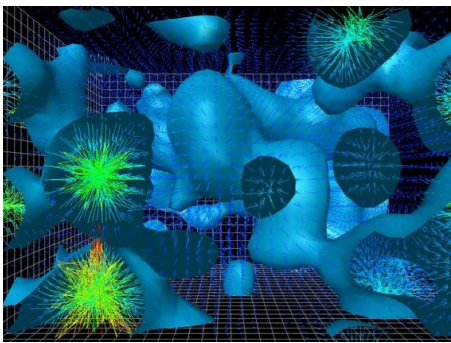


Nuclear force - no long range, only $\delta(r)$ and r^{-3} from π .

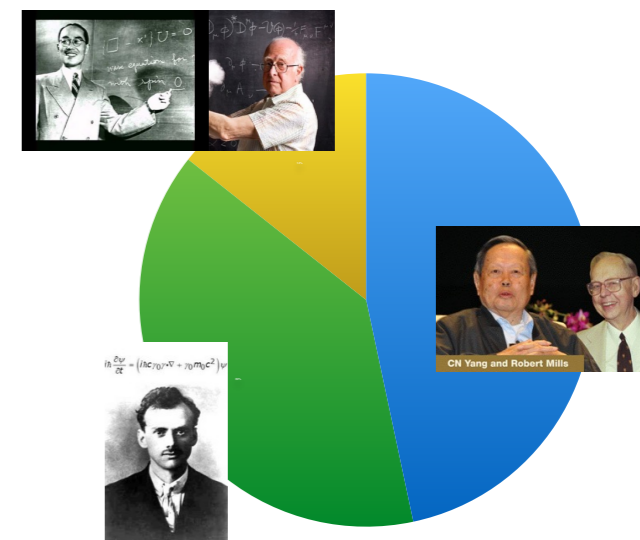
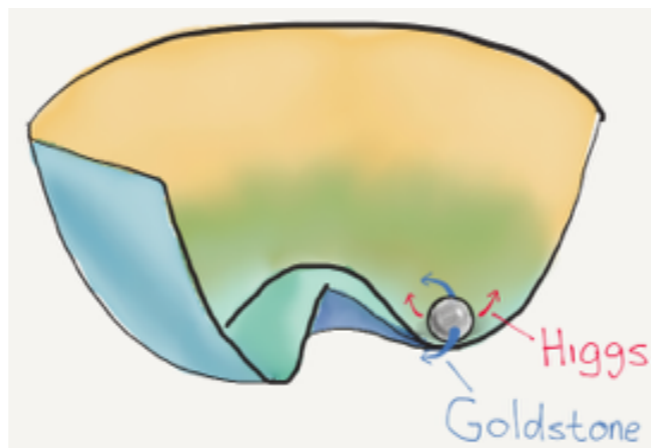
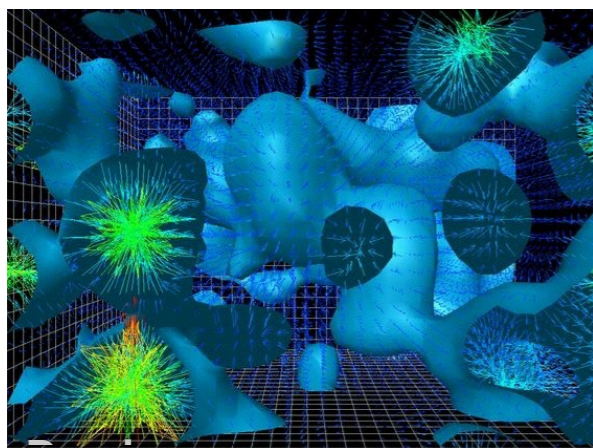
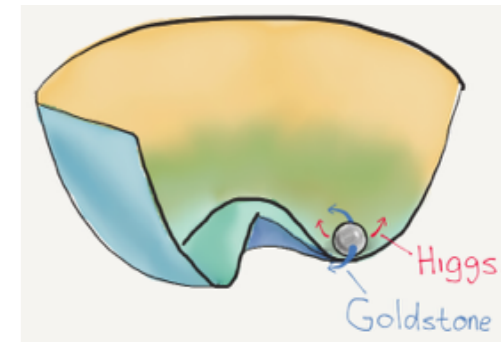
Short-range couplings modified

Radiation π 's need to be included

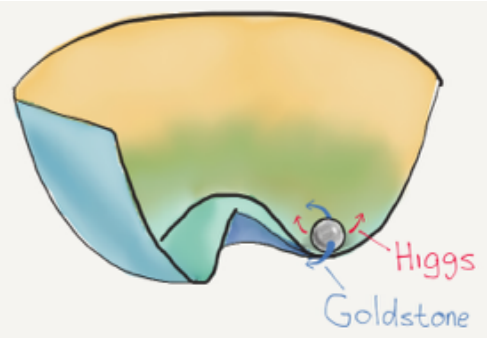
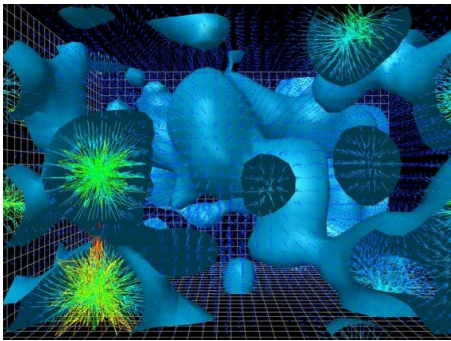
Currently do not know if nuclei exist near the chiral limit.



Summary



- Within the Standard Model, we “know” the mechanisms of mass generation, but have limited “understanding” of the QCD mechanism
- Quantitative decompositions for u,d quarks, and partially for s quarks
- What is the optimal experimental program to pursue?
 - Needs more thought/effort
- Challenge to determine nucleon mass when $v_{ev}=0$
 - How much does the Higgs mechanism contribute?



FIN